Reasoning about Natural Strategic Ability

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Preface

System Correctness

- A very important problem in critical systems:
  - Safety: errors can cost lives (e.g. Therac-25).
  - Mission: errors can cost in terms of objectives (e.g. Arianne 5).
  - Business: failure can cost in loss of money (e.g. Denver airport).
- In such systems failure is not an option.

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Formal verification

- It studies in depth system correctness.
- It is based on mathematic/logical methods.
- One of the most important contributions is the Model Checking.

Model checking (1)

$M \models \varphi$

There are three fundamental parts:

- M : modeling a system;
- φ : specifying a property;
- $\models$: verifying that the model M satisfies the property φ.

Model checking (2)

Model

A Kripke Structure is a tuple $K = \langle \mathrm{AP}, \mathrm{St}, S_0, R, L \rangle$, where:

- AP is a set of atomic propositions;
- St is a set of states;
- $S_0 \subseteq \mathrm{St}$ is a set of initial states;
- $R \subseteq \mathrm{St} \times \mathrm{St}$ is a transition relation;
- $L : \mathrm{St} \to 2^{\mathrm{AP}}$ is a labeling function.

Model checking (3)

Specification

- Temporal logics allow to describe the order of the events without define the time explicitly.
- The main temporal logics:
    - LTL [Pnu77](Linear Temporal Logic), in a computation the events are totally ordered.
    - CTL [CE81] (Computation Tree Logic), in a computation the events are partially ordered.

Model checking (4)

Verification

Given a Kripke structure K and a specification φ, the problem of the model checking consists to verify if $K, s \models \varphi$, for each initial state s.

Open Systems (1)

Key aspects

- There are many agents (players) interacting among them.
- Each agent has a set of *strategies*.
- A *strategy* is a conditional plan that at each step of the game prescribes an action.
- The composition of strategies, one for each player, induces an unique computation.

Open Systems (2)

Model

- A game structure is a tuple $M =< AP, St, s_I, Ac, Ag, tr >$, where:
  - AP is a set of atomic propositions;
  - St is a set of states;
  - $s_I \in S$ is a designated initial state;
  - Ac is a set of actions;
  - Ag is a set of agents;
  - tr is a transition function.
- Depending on the interaction between the agents:
  - *Turn-based* $\Rightarrow$ The states of the game are partitioned between the agents, then the owner of a state determines the move to take and thus the next state of the game.
  - *Concurrent* $\Rightarrow$ The agents choose a move (i.e, actions) simultaneously and independently, and the choices together determine the next state of the game.

Open Systems (3)

Plays and Histories

- The infinite paths starting from s_I represent all possible plays.
- The finite paths starting from s_I represent all possible histories.
- The set of all histories is denoted by H.

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Strategies

- Depending on the memory, we distinguish between:
  - *memoryless strategies (r)* $\Rightarrow \sigma : St \to \mathrm{Ac}$;
  - *memoryfull strategies (R)* $\Rightarrow \sigma : H \to \mathrm{Ac}$.
- In r case, the players take a decision by considering the actual state of the game.
- In R case, the players take a decision by considering the history of the game.

Example: paper, rock, and scissor

- $\mathrm{Ag} = \{\mathsf{A}\colon \mathit{Alice}, \mathsf{B}\colon \mathit{Bob}\}$
- $\mathrm{Ac} = \{\mathsf{P}\colon \mathit{Paper}, \mathsf{R}\colon \mathit{Rock}, \mathsf{S}\colon \mathit{Scissor}\}$

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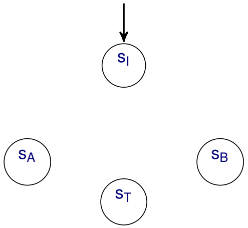

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- s_I: *initial state*

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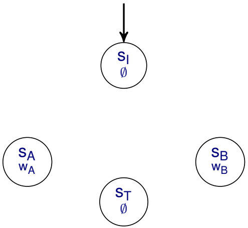

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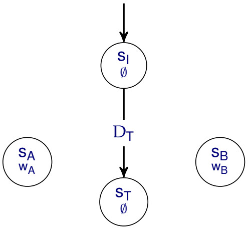

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- $D_T = \{(P, P), (R, R), (S, S)\}$

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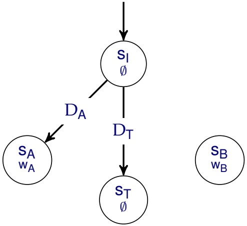

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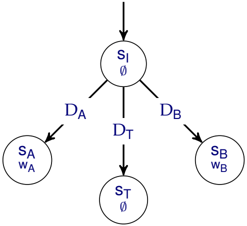

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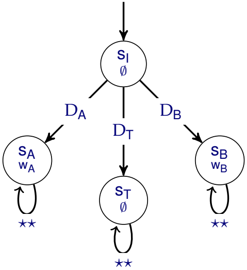

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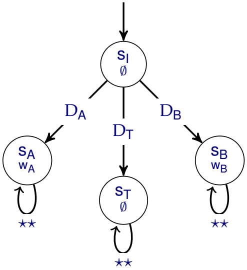

- A possible play is $\pi = s_I \cdot (s_A)^\omega$.

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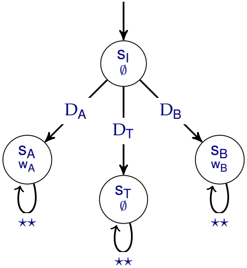

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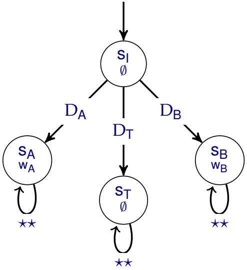

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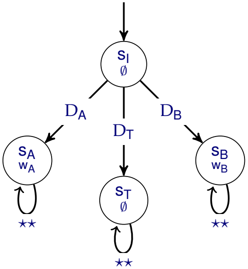

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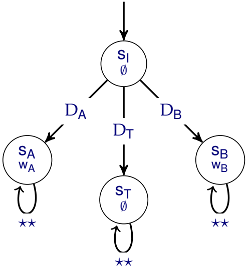

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- An example of history is $h = s_I$.
- A strategy for A is $\sigma_A(h) = \mathsf{P}$, $\forall h \in H$.
- A strategy for B is $\sigma_B(h) = \mathsf{R}$, $\forall h \in H$.
- σ_A and σ_B induce the play π.

Open Systems (4)

Specification

- *Internal* $\Rightarrow$ directly over the game structure (Examples: Reachability, Safety, Nash Equilibrium).
- *External* $\Rightarrow$ logics for the strategic reasoning (Examples: ATL [AHK02], Strategic Logic [MMV10]).

Open Systems Specification (1)

Alternating-Time Temporal Logic (ATL*) [AHK02]

- ATL* generalizes CTL* by replacing the path quantification with strategy quantification over coalition of agents.
- It uses the strategy modality $\langle\langle A \rangle\rangle$ and $[\![A]\!]$, with A set of agents.
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Example

$\langle\langle \{\alpha, \beta\} \rangle\rangle \mathsf{G} \neg \mathit{fail}$: the agents α and β have a collective strategy such that the system does not reach the failure state, independently from the strategies of the other agents.

Open Systems Specification (2)

Model checking complexities

- ATL^* with:
  - Memoryless strategies is $PSPACE - complete$ [Sch04].
  - Memoryfull strategies is $2EXPTIME - complete$ [AHK02].
- ATL with:
  - Memoryless strategies is $PTIME - complete$ [AHK02].
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Between Mathematics and Real Life

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- However, strategies are *mathematical creatures* $\Longrightarrow$ *any function* from system states to actions is fine.
- This makes sense for robots or programs, but not for humans!
- Strategies for humans should be simple in order for the person to *understand* it, *memorize* it, and *execute* it.

Natural Strategies [JMM17]

A *natural memoryless strategy* s_a for agent a is a *list of condition-action rules*

$$(cond, act)$$

such that:

- $cond$ is a boolean combination of propositions,
- act is an available action in every state $q \models cond$,
- the last pair on the list is $(\top, idle)$.

Natural Strategies: Example

Consider the following strategy for *buying a train ticket*:

1. $(\neg\mathsf{ticket} \wedge \neg\mathsf{selected},\ \mathit{select})$;
2. $(\neg\mathsf{ticket} \wedge \mathsf{selected},\ \mathit{pay})$;
3. $(\top,\ \mathit{idle})$.

NATURAL STRATEGIES: COMPLEXITY

The complexity of strategy s_a ($compl(s_a)$) can be defined by:

- Number of used propositions $\Rightarrow |dom(s_a)|$;
- Largest condition $\Rightarrow \max\{|\phi| \mid (\phi, \alpha) \in s_a\}$;
- Total size of the representation $\Rightarrow \sum_{(\phi,\alpha)\in s_a} |\phi|$.

Reasoning about Natural Ability: NatATL

Syntax

A formula in NatATL is defined as:

$$\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A \rangle\rangle^{\leq k} X\varphi \mid \langle\langle A \rangle\rangle^{\leq k} \varphi U \varphi \mid \langle\langle A \rangle\rangle^{\leq k} \varphi W \varphi.$$

where $\mathsf{p} \in AP$, $k \in \mathbb{N}$, and A is a set of agents.

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where $\mathsf{p} \in AP$, $k \in \mathbb{N}$, and A is a set of agents.

Semantics

$M, q \models \langle\langle A \rangle\rangle^{\leq k}\gamma$ iff there is a natural strategy s_A such that $compl(s_A) \leq k$, and for each path $\lambda \in out(q, s_A)$ we have $M, \lambda \models \gamma$.

What's the Use?

Reasoning about usability, example: *ticket vending machine*

- It is not enough that a customer has a strategy to buy the ticket ($\langle\langle c\rangle\rangle F\mathsf{buy}$).
- If the strategy is too complex, people won't use it anyway.
- Instead, we should require $\langle\langle c\rangle\rangle^{\leq k} F\mathsf{buy}$ for a reasonably low k.

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Gaming

- The designer can define the *game level* by the *complexity of the smallest winning strategy* for the player.
- Formally, the level k iff $\langle\langle a\rangle\rangle^{\leq k}F\mathsf{win} \wedge \neg\langle\langle a\rangle\rangle^{\leq k-1}F\mathsf{win}$.

Natural Strategies with Recall

- Similar to memoryless strategies, but the conditions are given by *regular expressions* over Boolean formulas.
- Example: a strategy for a *Wild West explorer*:
  1. $(\mathsf{safe}^*, \textit{digGold})$;
  2. $(\mathsf{safe}^* \cdot (\neg\mathsf{safe} \wedge \mathsf{haveGun}), \textit{shoot})$;
  3. $(\mathsf{safe}^* \cdot (\neg\mathsf{safe} \wedge \neg\mathsf{haveGun}), \textit{run})$;
  4. $(\top^* \cdot (\neg\mathsf{safe}) \cdot (\neg\mathsf{safe}), \textit{hide})$;
  5. $(\top^*, \textit{idle})$.

Relationships between Types of Natural Strategies (1)

Theorem

The following results hold in NatATL:

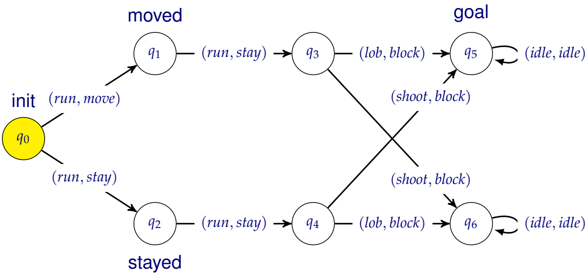
1. *For all M, q, and all formulas $\varphi = \langle\langle A \rangle\rangle^{\leq k}\gamma$, it holds that:*

$$M, q \models_r \varphi \text{ implies } M, q \models_R \varphi$$

2. *There exist M, q, and a formula $\varphi = \langle\langle A \rangle\rangle^{\leq k}\gamma$, such that:*

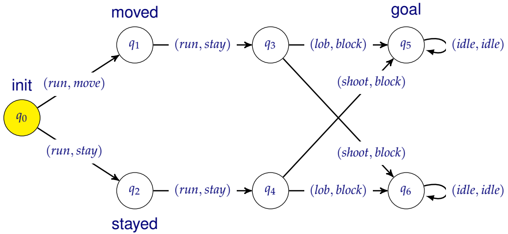
$$M, q \models_R \varphi \text{ and not } M, q \models_r \varphi$$

Example: Soccer scenario (1)



- Robot 1 is running towards the goal with the ball.
- The goalkeeper (robot 2) can either stay close to the goal line or move towards the attacker.
- Then, after one more step, the attacker can either shoot straight or lob the ball over the goalkeeper.

Example: Soccer scenario (2)

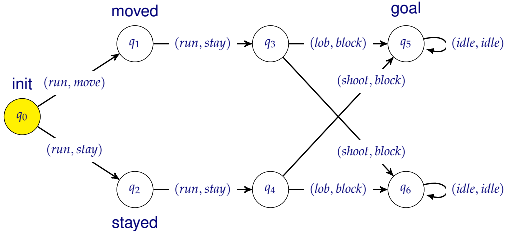


A strategy with recall for the attacker to score the goal can be:

1. (init, run);
2. (init $\cdot$ (moved $\vee$ stayed), run);
3. ($\top^* \cdot$ moved $\cdot \top$, lob);
4. ($\top^* \cdot$ stayed $\cdot \top$, $shoot$);
5. ($\top^*$, $idle$).

The complexity of the strategy is 22.

Example: Soccer scenario (3)



- Then, $\varphi = \langle\langle 1 \rangle\rangle^{\leq 22} F\mathsf{goal}$ is true for strategies with recall.
- On the other hand, φ is false for memoryless strategies.
- In fact, the formula is false for any bound k.
- To see that, recall that conditions in natural memoryless strategies can only refer to boolean properties of the current state.
- Then, it is impossible to define two different behaviors in states q_3 and q_4 within a natural memoryless strategy.

VERIFICATION OF NATURAL STRATEGIES

Model checking $NatATL_r$

- $\mathbf{P}$ for fixed or bounded k;
- $\mathbf{P}^{\mathbf{NP}} = \Delta_2^{\mathbf{P}}$-complete when k is a parameter of the problem.

Verification of Natural Strategies

Model checking $NatATL_r$

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Model checking $NatATL_R$

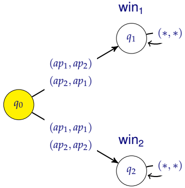
- $\Delta_2^{\mathbf{P}}$ for fixed or bounded k;
- $\mathbf{P^{NP^{NP}}} = \Delta_3^{\mathbf{P}}$ when k is a parameter of the problem.

Concurrent game with objectives [JMM]

A *concurrent game* is a tuple $G = (M, q_0, \Phi)$, where:

- M is a concurrent game structure,
- $q_0 \in St$ is a state in M,
- $\Phi : Ag \to \mathcal{L}_{LTL}$ assigns each agent with an LTL formula.

Example: Simple market scenario



Establ. Comp. \ New Comp.	ap_1	ap_2
ap_1	0, 1	1, 0
ap_2	1, 0	0, 1

- An established company (EC) and a new company (NC) have to choose the appearance for a product.
- Each company can choose between two different appearances for the product (ap_1 and ap_2).
- EC prefers the two products to look different.
- NC is better off when the products look alike.
- $\Phi_{EC} = F\mathsf{win}_1$ and $\Phi_{NC} = F\mathsf{win}_2$.

Decision problems: Surely Winning (1)

Definition

Given a concurrent game G, a subset of agents $A \subseteq Ag$, a natural number $k \in \mathbb{N}$, and a natural collective strategy s_A of A, we say that:

$$s_A \text{ is } \textit{surely winning} \text{ in } G \Leftrightarrow \forall \lambda \in out(q_0, s_A) \text{ and } a \in A\text{: } \lambda \models \Phi_a$$

Moreover, coalition A *surely wins* in G under bound k iff it has a sure winning strategy of size at most k.

Decision problems: Surely Winning (2)

Algorithm $SureWin(G, A, k)$:

$s_A = GuessStrat(G, A, k)$;
Prune M according to s_A, obtaining model M';
return $mCheck_{CTL^*}(M', q_0, \mathbf{A} \bigwedge_{i \in A} \Phi_i)$;

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Hint for lower bound

We show a reduction from model checking LTL.

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Complexity

SureWin is $PSPACE - complete$.

Decision problems: Nash Equilibrium (1)

Definition

Given a concurrent game G and a profile $s_{Ag} = (s_1, \ldots, s_i, \ldots, s_{|Ag|})$ of natural strategies under bound $k \in \mathbb{N}$:

s_{Ag} is a *Nash Equilibrium* in $G \Leftrightarrow \forall i \in Ag$, s_i is a best response.

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Best response

Given G, a player i, and a profile $s_{Ag} = (s_1, \ldots, s_i, \ldots, s_{|Ag|})$ under bound $k \in \mathbb{N}$, s_i is a *best response* in s_{Ag} if and only if:

$$path(s_{Ag}) \not\models \Phi_i \Rightarrow path((s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_{|Ag|})) \not\models \Phi_i$$

for all $s'_i \in \Sigma^r_i$ such that $compl(s'_i) \leq k$.

Decision problems: Nash Equilibrium (2)

Algorithm $\mathit{IsNotNash}(G, s_{Ag}, k)$:

for every $i \in Ag$ do
  if $\mathit{path}(s_{Ag}) \not\models \Phi_i$ then $s'_i = \mathit{GuessStrat}(G, i, k)$;
    if $\mathit{path}((s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_{|Ag|})) \models \Phi_i$ then return(true);
return (false);

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Hint for lower bound

We use a reduction from SAT.

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Algorithm $IsNotNash(G, s_{Ag}, k)$:

for every $i \in Ag$ do
if $path(s_{Ag}) \not\models \Phi_i$ then $s'_i = GuessStrat(G, i, k)$;
if $path((s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_{|Ag|})) \models \Phi_i$ then return(true);
return (false);

Hint for lower bound

We use a reduction from SAT.

Complexity

IsNotNash is $\mathbf{NP} - complete \Rightarrow$ *IsNash* is $co - \mathbf{NP} - complete$.

Decision problems: Nash Equilibrium (3)

Algorithm $\mathit{WinsSomeNash}(G, i, k)$:

$s_{Ag} = \mathit{GuessStrat}(G, Ag, k)$;
if $\mathit{path}(s_{Ag}) \models \Phi_i$ and not $\mathit{IsNotNash}(G, s_{Ag}, k)$ then return (true);
else return (false);

Decision problems: Nash Equilibrium (3)

Algorithm *WinsSomeNash*(G, i, k):

$s_{Ag} = GuessStrat(G, Ag, k)$;
if $path(s_{Ag}) \models \Phi_i$ and not $IsNotNash(G, s_{Ag}, k)$ then return (true);
else return (false);

Hint for lower bound

We use a reduction from QBF_2.

Decision problems: Nash Equilibrium (3)

Algorithm $WinsSomeNash(G, i, k)$:

$s_{Ag} = GuessStrat(G, Ag, k)$;
if $path(s_{Ag}) \models \Phi_i$ and not $IsNotNash(G, s_{Ag}, k)$ then return (true);
else return (false);

Hint for lower bound

We use a reduction from QBF_2.

Complexity

WinsSomeNash is $\mathbf{NP}^{\mathbf{NP}} = \Sigma_2^{\mathbf{P}}$-complete.

Decision problems: Nash Equilibrium (4)

Algorithm *LosesSomeNash*(G, i, k):

$s_{Ag} = GuessStrat(G, Ag, k)$;
if $path(s_{Ag}) \not\models \Phi_i$ and not $IsNotNash(G, s_{Ag}, k)$ then return (true);
else return (false);

Decision problems: Nash Equilibrium (4)

Algorithm *LosesSomeNash*(G, i, k):

$s_{Ag} = GuessStrat(G, Ag, k)$;
if $path(s_{Ag}) \not\models \Phi_i$ and not $IsNotNash(G, s_{Ag}, k)$ then return (true);
else return (false);

Hint for lower bound

We use a reduction from the complement problem of *WinsSomeNash*.

Decision problems: Nash Equilibrium (4)

Algorithm *LosesSomeNash*(G, i, k):

$s_{Ag} = GuessStrat(G, Ag, k)$;
if $path(s_{Ag}) \not\models \Phi_i$ and not $IsNotNash(G, s_{Ag}, k)$ then return (true);
else return (false);

Hint for lower bound

We use a reduction from the complement problem of *WinsSomeNash*.

Complexity

WinsAllNash is $co - \mathbf{NP}^{\mathbf{NP}} = \Pi_2^{\mathbf{P}}$-complete.

Conclusions

- We proposed the concept of *natural strategies*, based on an intuitive representation of conditional plans.
- We proposed how to measure the complexity of such strategies.
- We defined NatATL, a variant of alternating-time temporal logic to reason about natural strategic ability.
- We studied the complexity of NatATL model checking.
- We considered two main cases here: memoryless strategies and strategies with recall of the past.
- We showed that the relationship between natural strategies with recall and memoryless is more intricate than normally in *ATL*.
- We investigated some decision problems for natural abilities of agents in concurrent games with *LTL* winning conditions.

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