# Transforming Coloured Petri Nets to Counter systems for Parametric Verification:

## *A Stop-and-Wait Protocol Case study*

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# Motivation

- analysis of network protocols
- often modelled using (coloured) Petri nets
- state space explosion $\Rightarrow$ difficult to analyse
- parametric models

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# Motivation

- analysis of network protocols
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- state space explosion ⇒ difficult to analyse
- parametric models

⇒ use :

- acceleration techniques to cope with the state space explosion problem
- FAST tool capabilities for parametric analysis

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# Outline

- FAST tool
  - Counter systems
  - Acceleration technique
  - Input/output of FAST
- From Petri nets to counter systems
  - General technique
  - Handling coloured Petri nets
- Stop-and-wait Protocols
  - Coloured Petri net model
  - Counter system model
  - Analysis

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# Counter Systems

- automata (control graph)
- extended with a finite set of unbounded integer variables
- transitions labelled with:
  - a guard expressed in Presburger arithmetics
  - an action expressed as an affine function over the integer variables

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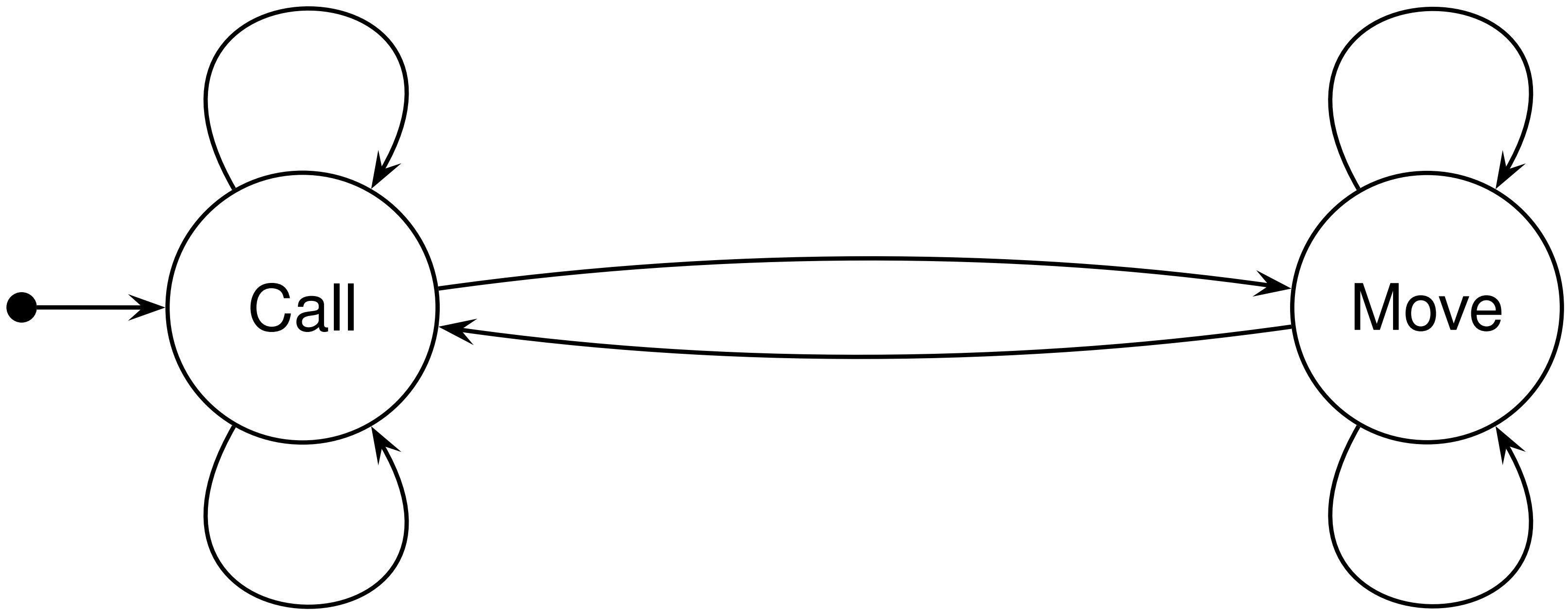
# Example



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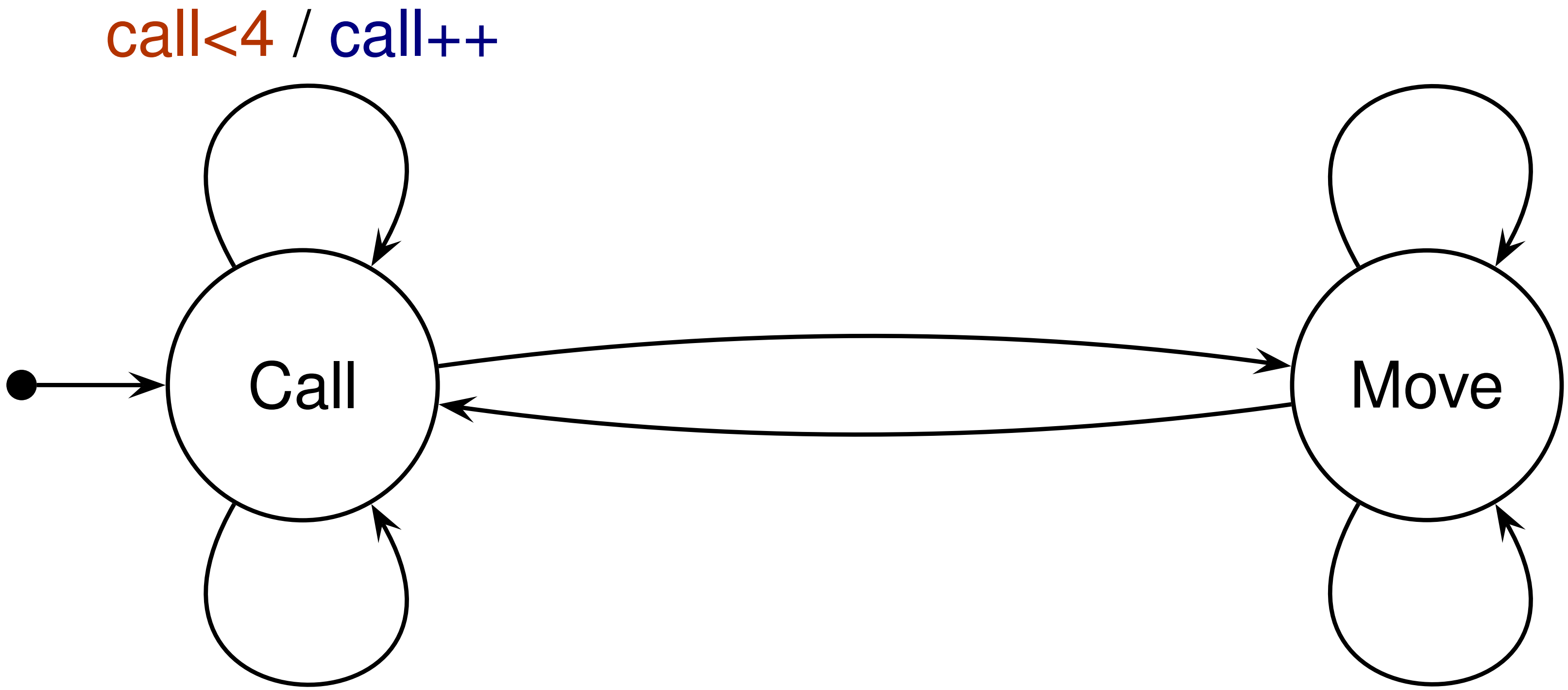

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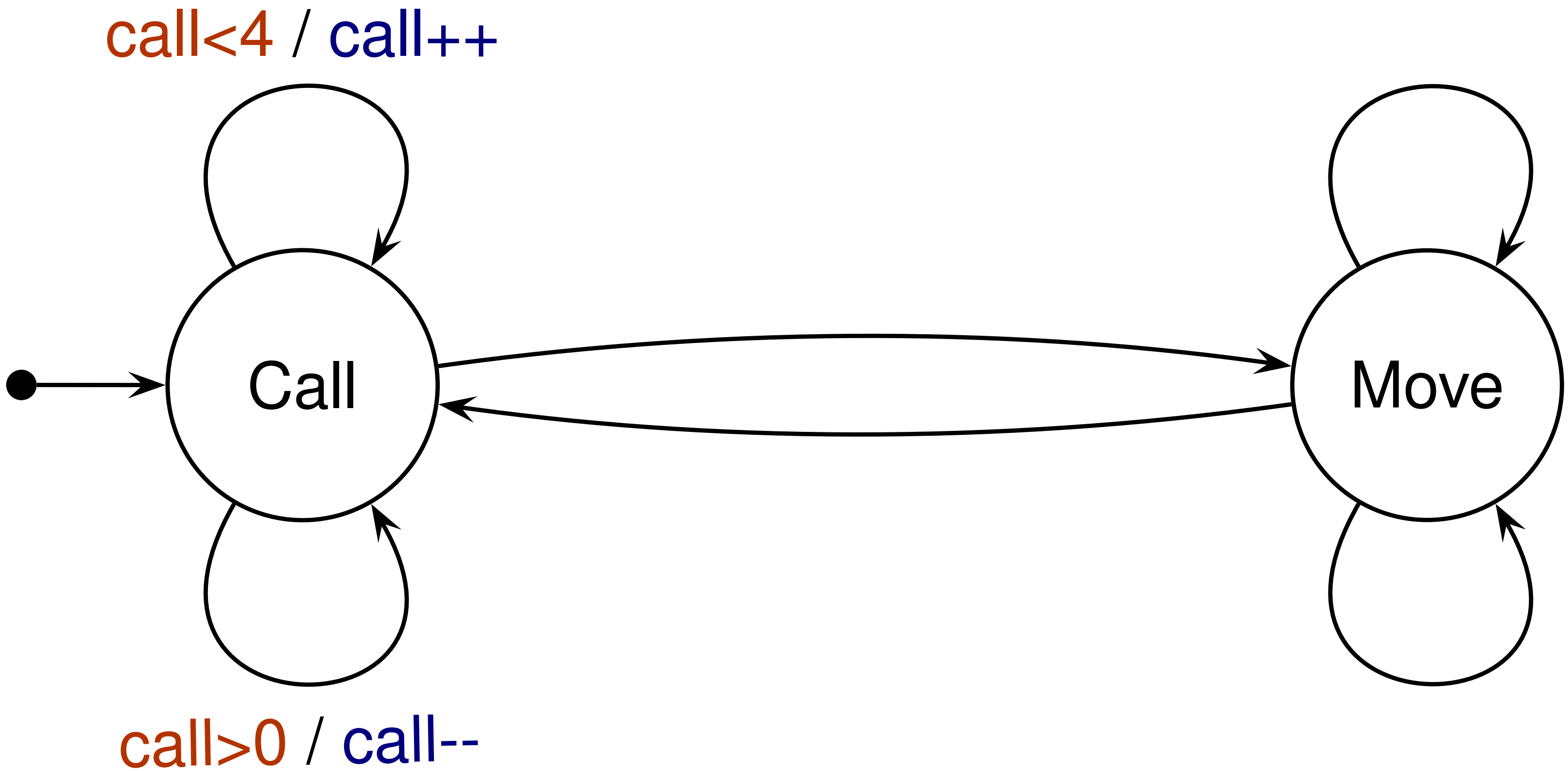

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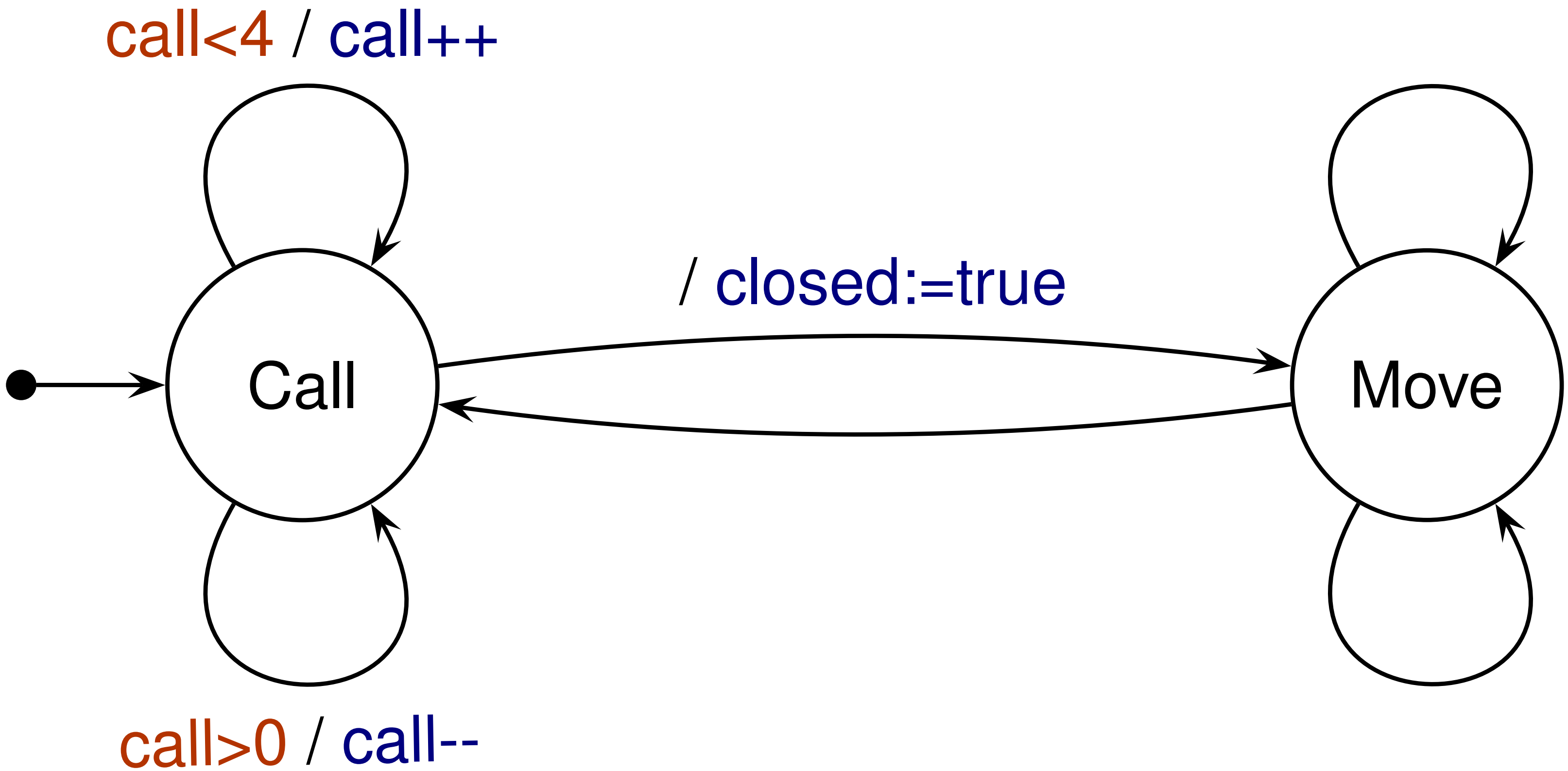

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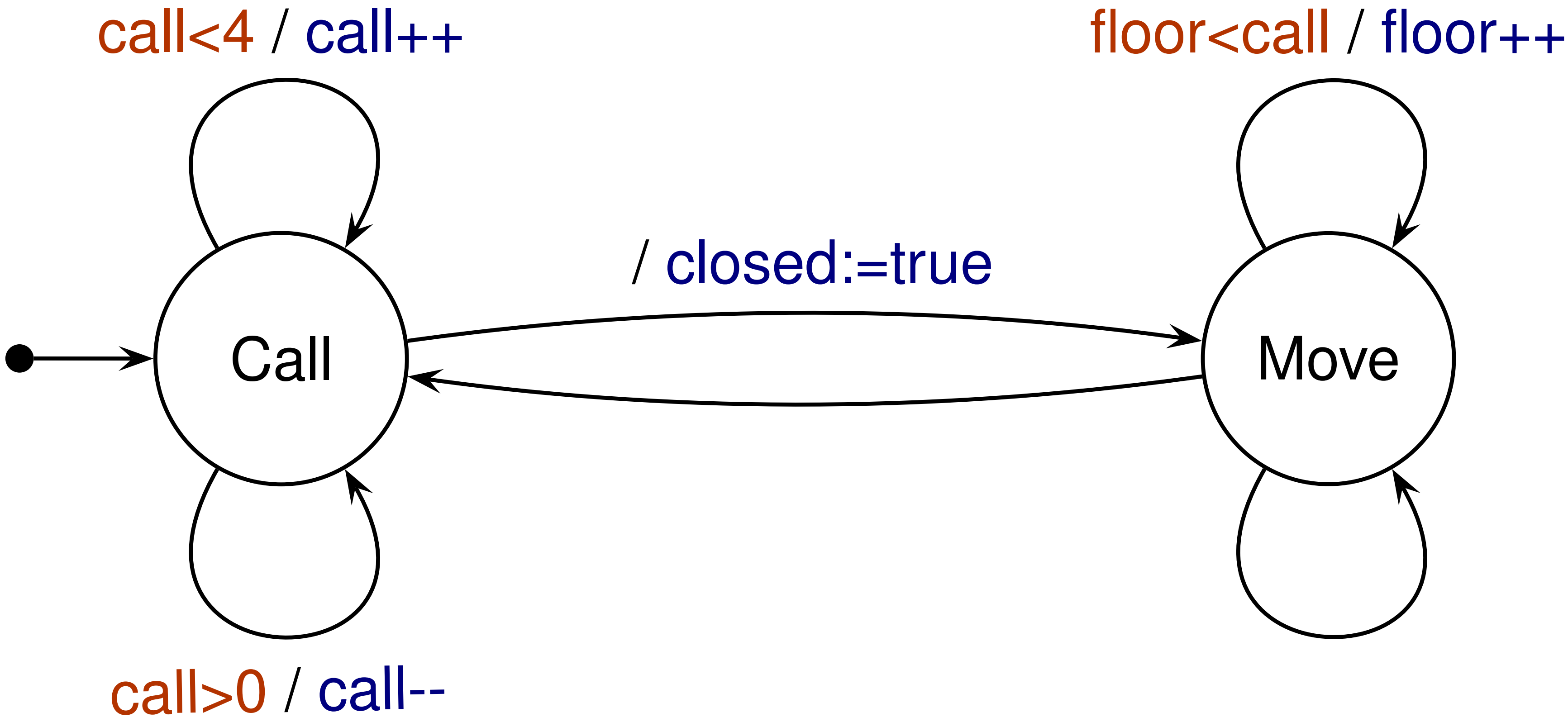
call<4 / call++
floor<call / floor++
/ closed:=true
Call
Move
call>0 / call--

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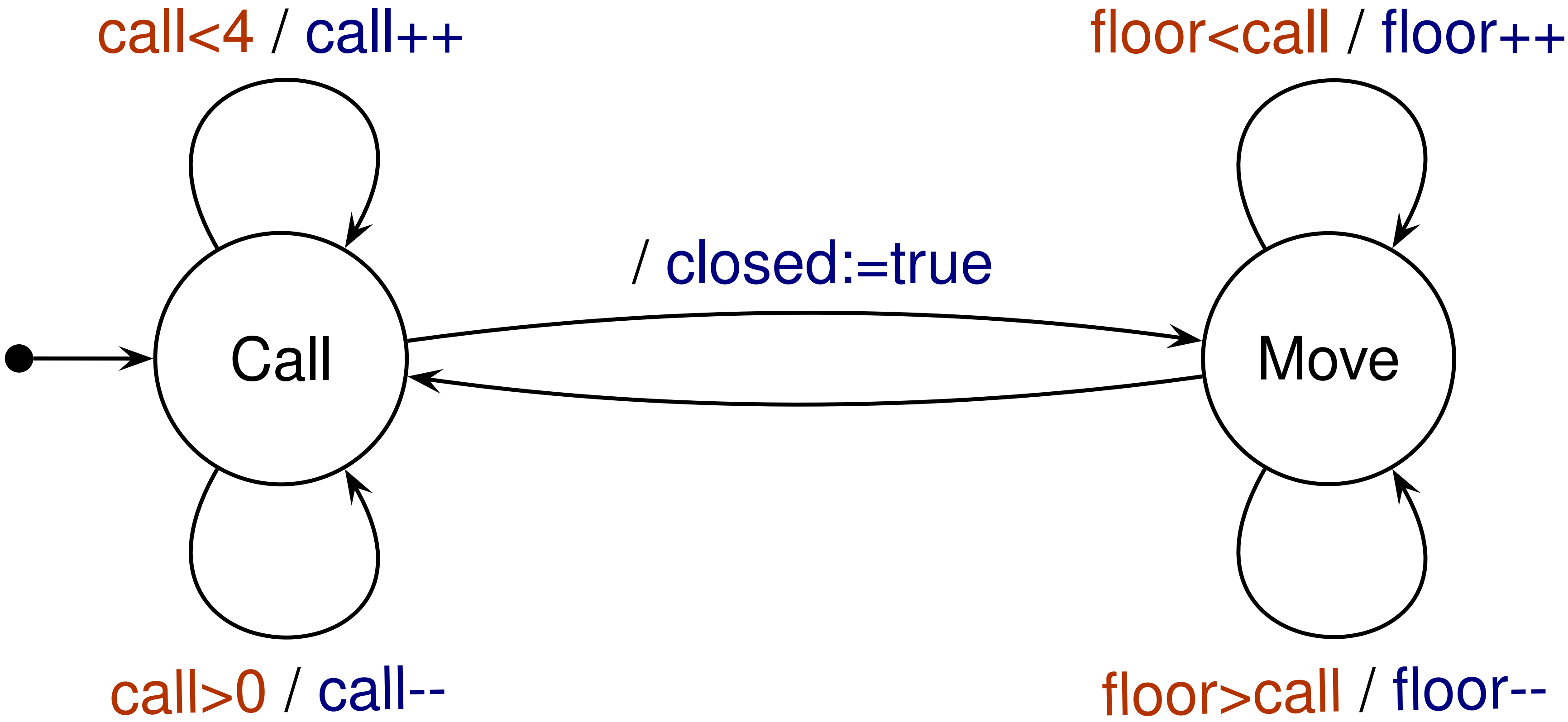
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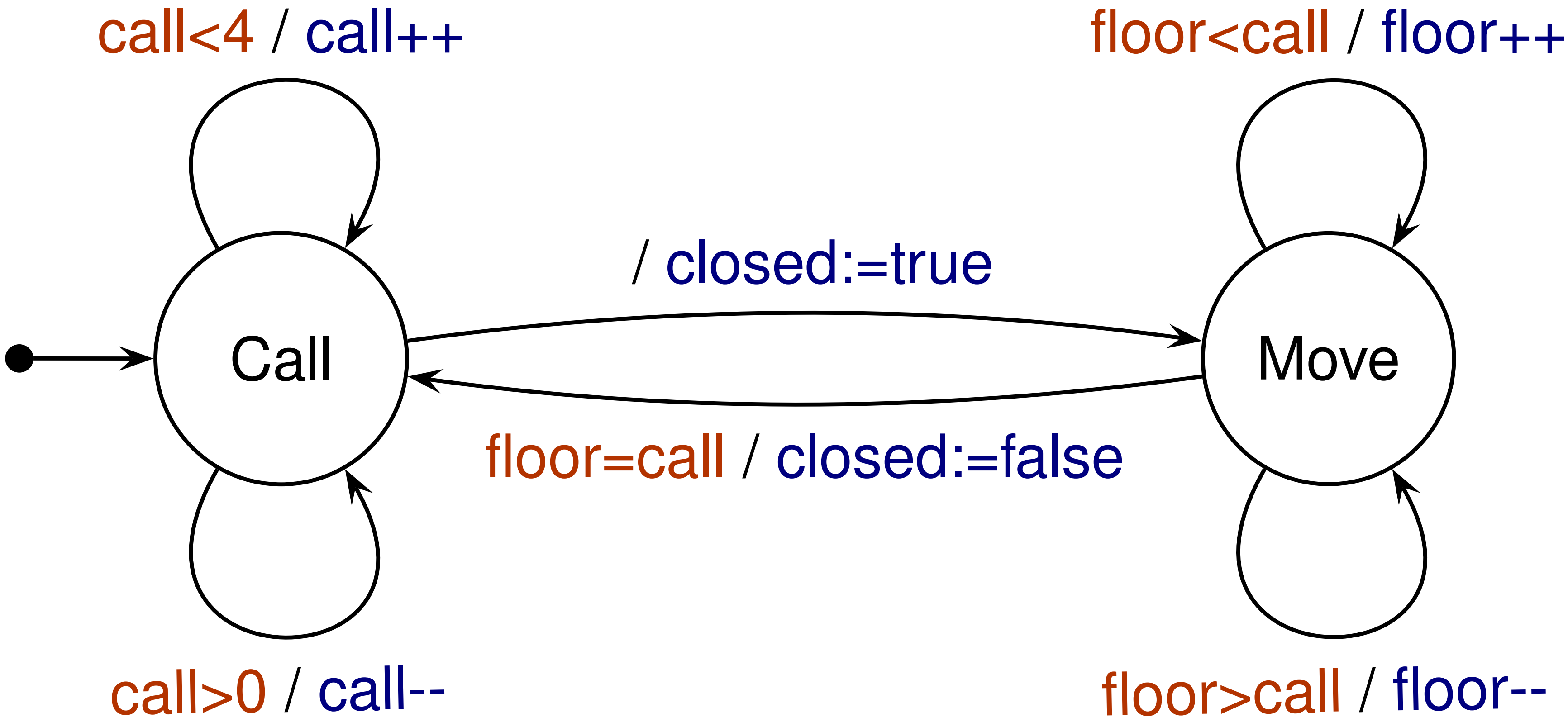
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- often infinite $\longrightarrow$ classical algorithm does not terminate
- $\Rightarrow$ use of acceleration techniques
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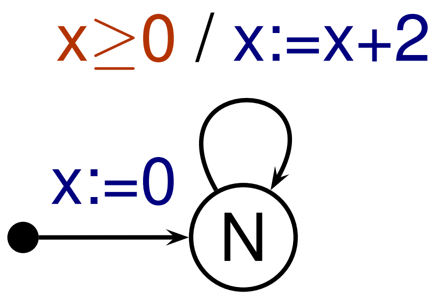

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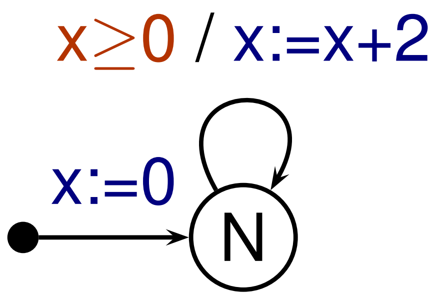


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$Reach \supseteq \{0\}$

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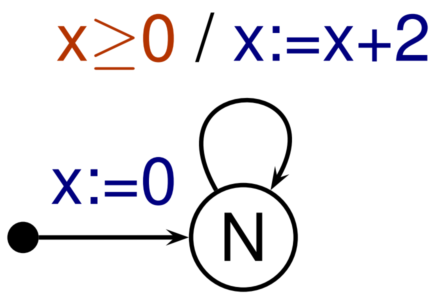


Classical algorithm:

$Reach \supseteq \{0, 2\}$

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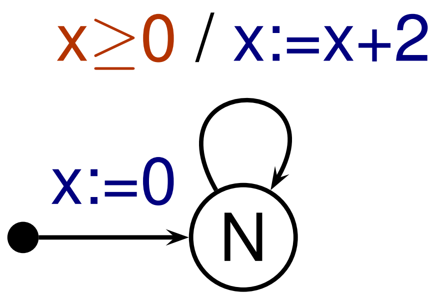


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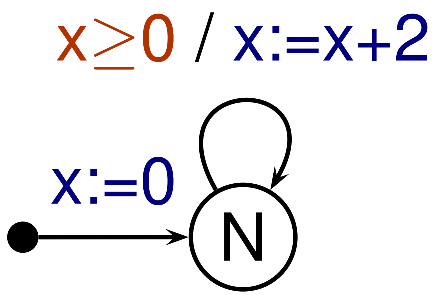


Acceleration:
$Reach$:=$2.\mathbb{N}$

# I/O of FAST

**model** : counter system

**strategy** : sequence of computations to check a safety property, described by a script language operating on:

- regions (sets of states)
- transitions
- booleans

and using operators to perform:

- sets and boolean operations
- forward/backward reachability

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# Petri Nets → Counter Systems

- a unique state
- one counter per place of the net
- one transition per transition of the net. Each transition:
  - loops onto the unique state
  - guard: enabling condition in the net
  - action: mimics the Petri net firing rule

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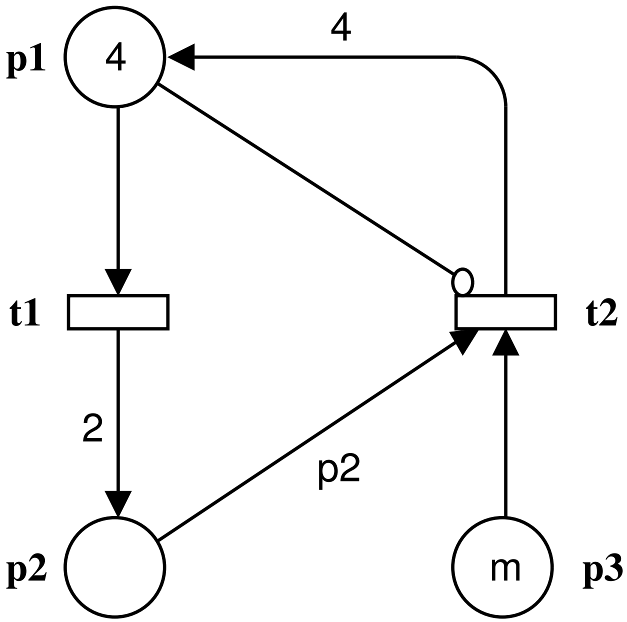
# Example




```
model n1 {
  var p1, p2, p3;
  states dummy;
  transition t1 := {
   from := dummy;
   to := dummy;
   guard := p1>=1;
   action := p1'=p1-1, p2'=p2+2;
  };
  transition t2 := {
   from := dummy;
   to := dummy;
   guard := p3>=1 && p1=0;
   action := p1'=p1+4, p2'=0, p3'=p3-1;
  };
}
```

# Handling CPNs

- often a single token (or none) in a place ⇒ represent colour set with an integer
- integers or enumerable types easy to map
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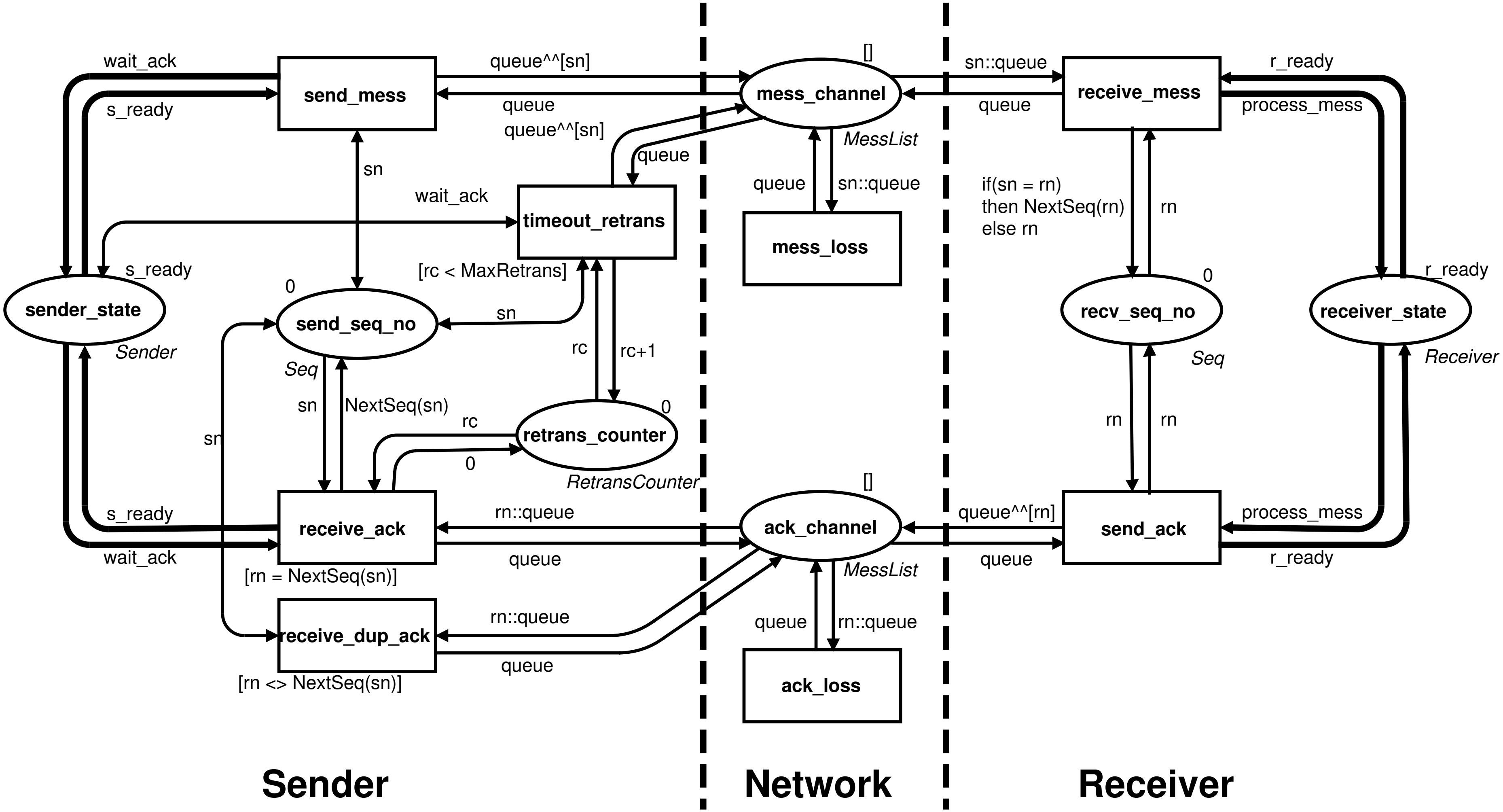
- often a single token (or none) in a place $\Rightarrow$ represent colour set with an integer
- integers or enumerable types easy to map
- queues are more complex
  - several types of messages but not simultaneously $\Rightarrow$ count the number of messages. One counter per type
  - at most two types of messages $a$ and $b$ at the same time in a FIFO queue, the queue being of the form $a^*b^* \Rightarrow 4$ variables:
    1. $a\_type$ type of messages $a$
    2. $nb\_a\_type$ number of messages of type $a$
    3. $b\_type$ type of messages $b$
    4. $nb\_b\_type$ number of messages of type $b$

# SWP CPN model



# SWP counter system model

```
var SState, SSeqNb, Retrans, MaxRetrans, RSeqNb, RState, MaxSeqNb,
   MCOld, MCNew, NbMCOld, NbMCNew, ACOld, ACNew, NbACOld, NbACNew;
```

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# SWP counter system model

```
var SState, SSeqNb, Retrans, MaxRetrans, RSeqNb, RState, MaxSeqNb,
   MCOld, MCNew, NbMCOld, NbMCNew, ACOld, ACNew, NbACOld, NbACNew;
states dummy;
```

# SWP counter system model

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   MCOld, MCNew, NbMCOld, NbMCNew, ACOld, ACNew, NbACOld, NbACNew;
states dummy;
transition sendM1 := {
   from := dummy;
   to := dummy;
   guard := SState=1 && NbMCOld=0;
   action := SState'=0,
    MCNew'=SSeqNb, NbMCNew'=1, MCOld'=SSeqNb, NbMCOld'=1;};
```

# SWP counter system model

```
var SState, SSeqNb, Retrans, MaxRetrans, RSeqNb, RState, MaxSeqNb,
   MCOld, MCNew, NbMCOld, NbMCNew, ACOld, ACNew, NbACOld, NbACNew;
states dummy;
transition sendM1 := {
   from := dummy;
   to := dummy;
   guard := SState=1 && NbMCOld=0;
   action := SState'=0,
    MCNew'=SSeqNb, NbMCNew'=1, MCOld'=SSeqNb, NbMCOld'=1;};
transition sendM2 := {
   from := dummy;
   to := dummy;
   guard := SState=1 && !(NbMCOld=0);
   action := SState'=0, MCNew'=SSeqNb, NbMCNew'=1;
};
...
```

# Analysis

```
strategy analyseSWP {
    setMaxState(0);
    setMaxAcc(0);
```

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strategy analyseSWP {
    setMaxState(0);
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    Transitions t := {sendM1, sendM2, ...};
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# Analysis

```
strategy analyseSWP {
   setMaxState(0);
   setMaxAcc(0);

   Transitions t := {sendM1, sendM2, ...};

   Region init := {state=dummy && SState=1 && SSeqNb=0 &&
     Retrans=0 && MCOld=0 && MCNew=0 && NbMCOld=0 && NbMCNew=0 &&
     ACOld=1 && ACNew=1 && NbACOld=0 && NbACNew=0 &&
     RSeqNb=0 && RState=1 && MaxSeqNb=5};
```

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# Analysis

```
strategy analyseSWP {
   setMaxState(0);
   setMaxAcc(0);

   Transitions t := {sendM1, sendM2, ...};

   Region init := {state=dummy && SState=1 && SSeqNb=0 &&
     Retrans=0 && MCOld=0 && MCNew=0 && NbMCOld=0 && NbMCNew=0 &&
     ACOld=1 && ACNew=1 && NbACOld=0 && NbACNew=0 &&
     RSeqNb=0 && RState=1 && MaxSeqNb=5};

   Region reach := post*(init, t, 2);
```

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# Properties

- Consecutive sequence numbers in messages buffer:

```
Region diffoldnewM := {(MCOld=MCNew) || (MCNew=MCOld+1) ||
                        (MCOld=MaxSeqNb && MCNew=0)};
```

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# Properties

- Consecutive sequence numbers in messages buffer:

```
Region diffoldnewM := {(MCOld=MCNew) || (MCNew=MCOld+1) ||
                        (MCOld=MaxSeqNb && MCNew=0)};

if (subSet(reach,diffoldnewM))
   then print("Consecutive nb in message buffer OK");
   else print("Consecutive nb in message buffer NOK");
endif
```

# Properties

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Region diffoldnewM := {(MCOld=MCNew) || (MCNew=MCOld+1) ||
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   then print("Consecutive nb in message buffer OK");
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endif
```

- Consecutive sequence numbers in acknowledgements buffer
- Modelling assumptions w.r.t. the queue are valid

# Properties

- Lowest upper bound in messages buffer `2.MaxRetrans+1`:

```
Region Mbound := {(MCOld=MCNew &&
                   NbMCOld<=MaxRetrans+MaxRetrans+1) ||
                  (!(MCOld=MCNew) &&
                   NbMCOld+NbMCNew<=MaxRetrans+MaxRetrans+1)};
```

# Properties

- Lowest upper bound in messages buffer `2.MaxRetrans+1`:

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Region Mbound := {(MCOld=MCNew &&
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                  (!(MCOld=MCNew) &&
                   NbMCOld+NbMCNew<=MaxRetrans+MaxRetrans+1)};

if (subSet(reach,Mbound))
   then print("Mbound OK");
   else print("Mbound NOK");
endif
```

# Properties

- Lowest upper bound in messages buffer `2.MaxRetrans+1`:

```
Region Mbound := {(MCOld=MCNew &&
                   NbMCOld<=MaxRetrans+MaxRetrans+1) ||
                  (!(MCOld=MCNew) &&
                   NbMCOld+NbMCNew<=MaxRetrans+MaxRetrans+1)};

if (subSet(reach,Mbound))
   then print("Mbound OK");
   else print("Mbound NOK");
endif
```

- Lowest upper bound in acknowledgements buffer `2.MaxRetrans+1`
- Lowest upper bound in both buffers `2.MaxRetrans+1`

# Properties

- Stop and Wait property needs a bit of instrumentation: add a variable `SRprop` recording the number of the last message sent $+1$. Update it when sending a message, reset it when receiving the message. Then check that it is not possible to send a message if the previous one has not been received:

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# Properties

- Stop and Wait property needs a bit of instrumentation: add a variable SRprop recording the number of the last message sent $+1$. Update it when sending a message, reset it when receiving the message. Then check that it is not possible to send a message if the previous one has not been received:

```
if (isEmpty(reach && {SRprop>0 && SState=1}))
   then print("Send and then receive OK");
   else print("Send and then receive NOK");
endif
```

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# Properties

- Stop and Wait property needs a bit of instrumentation: add a variable `SRprop` recording the number of the last message sent $+1$. Update it when sending a message, reset it when receiving the message. Then check that it is not possible to send a message if the previous one has not been received:

```
if (isEmpty(reach && {SRprop>0 && SState=1}))
   then print("Send and then receive OK");
   else print("Send and then receive NOK");
endif
```

- Hence no loss except eventually the last message when `MaxRetrans` is reached

# Properties

No duplication: check that there is no state such that the receiver is ready to accept a new message with a sequence number different from the last message sent:

```
if (isEmpty(reach && {SRprop=MCOld+1 && RState=1 &&
                      NbMCOld>0 && !(MCOld=RSeqNb)}))
   then print("No duplication OK");
   else print("No duplication NOK");
endif
```

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# Properties

In sequence delivery: check that it is not possible to receive an original message with a sequence number different from the most recently sent:

```
if (isEmpty(reach && {RState=1 && NbMCOld>0 &&
                      MCMin=RSeqNb && !(SRprop=RSeqNb+1)}))
   then print("In sequence delivery OK");
   else print("In sequence delivery NOK");
endif
```

# Properties

- Deadlocks as expected:
  - `Retrans=MaxRetrans`
  - Sender not ready to send a new message: `SState=0`
  - both buffers empty: `MCOld=MCNew`, `ACOld=ACNew` and `NbMCOld=NbMCNew=NbACOld=NbACNew=0`

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# Conclusion

- parametric verification of stop-and-wait protocols with lossy or lossless channels
- verification of many properties
- translation of some CPNs with queues into counter systems

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