Building a Symbolic Model Checker from Formal Language Description

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UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

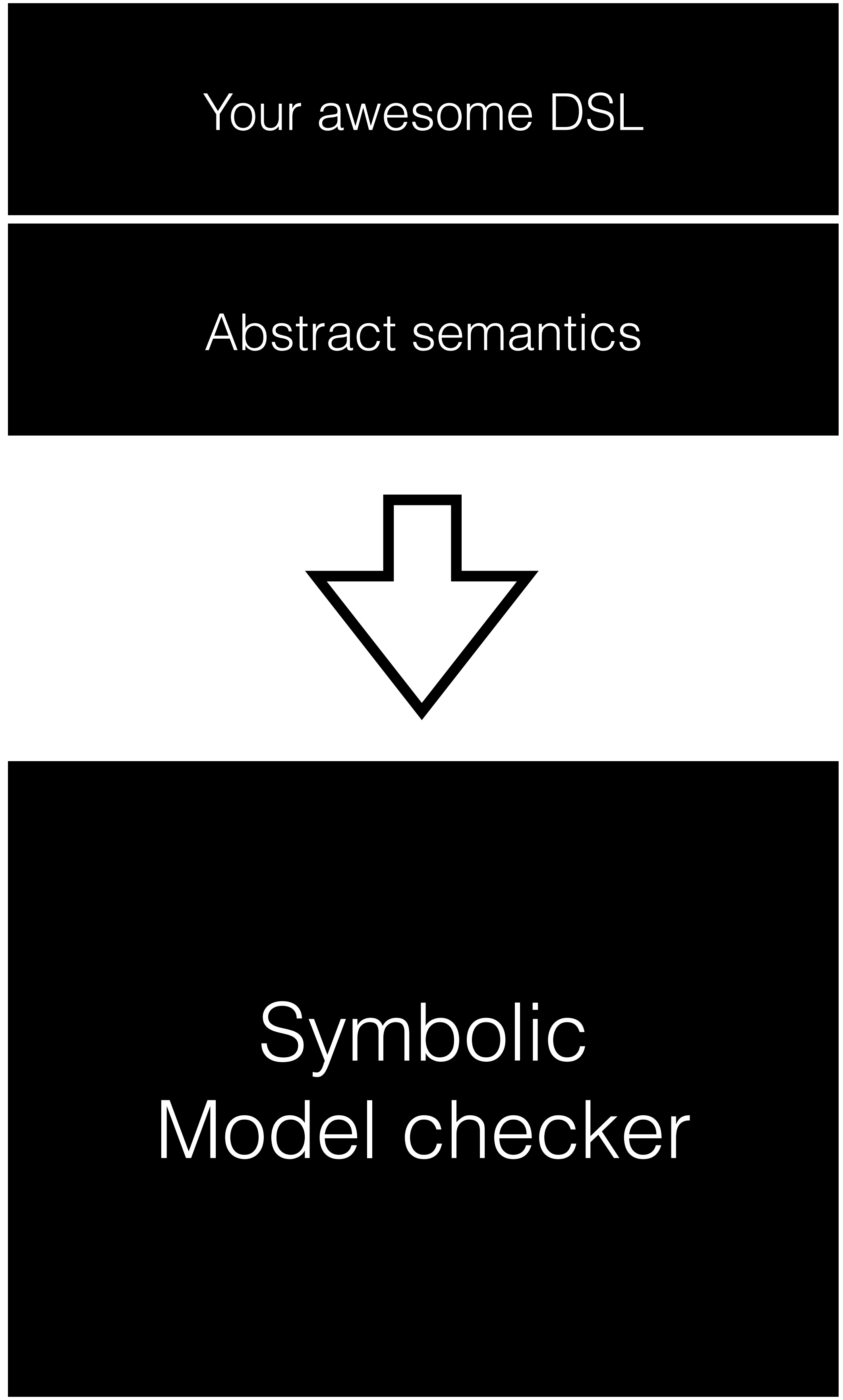
Méthodes Formelles pour les Systèmes Logiciels et Matériels

SMV

Your awesome DSL

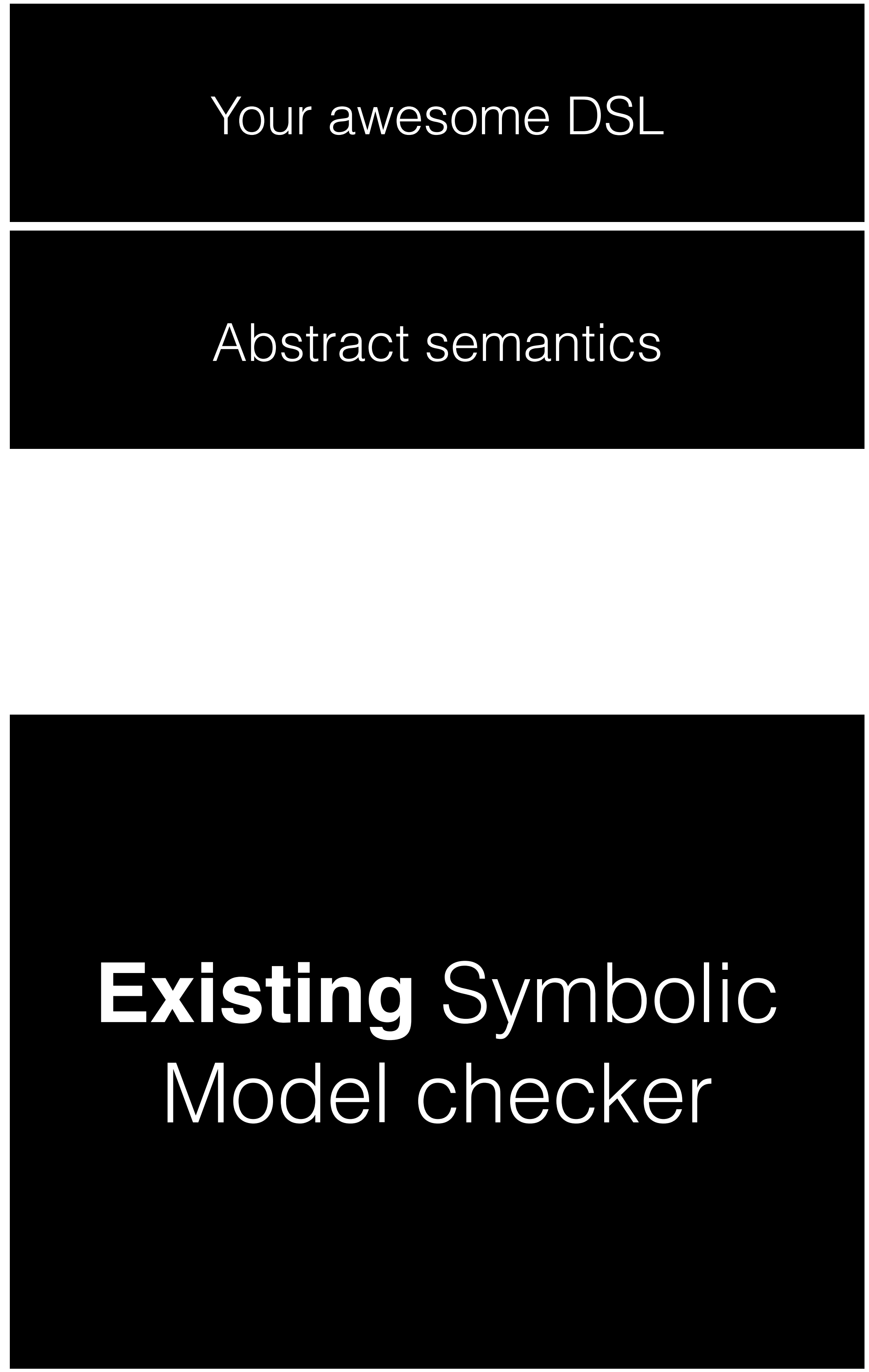
Your awesome DSL

Abstract semantics


Your awesome DSL
Abstract semantics
Symbolic
Model checker

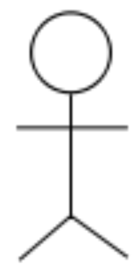
Your awesome DSL

Abstract semantics


Your awesome DSL
Abstract semantics
Existing Symbolic
Model checker

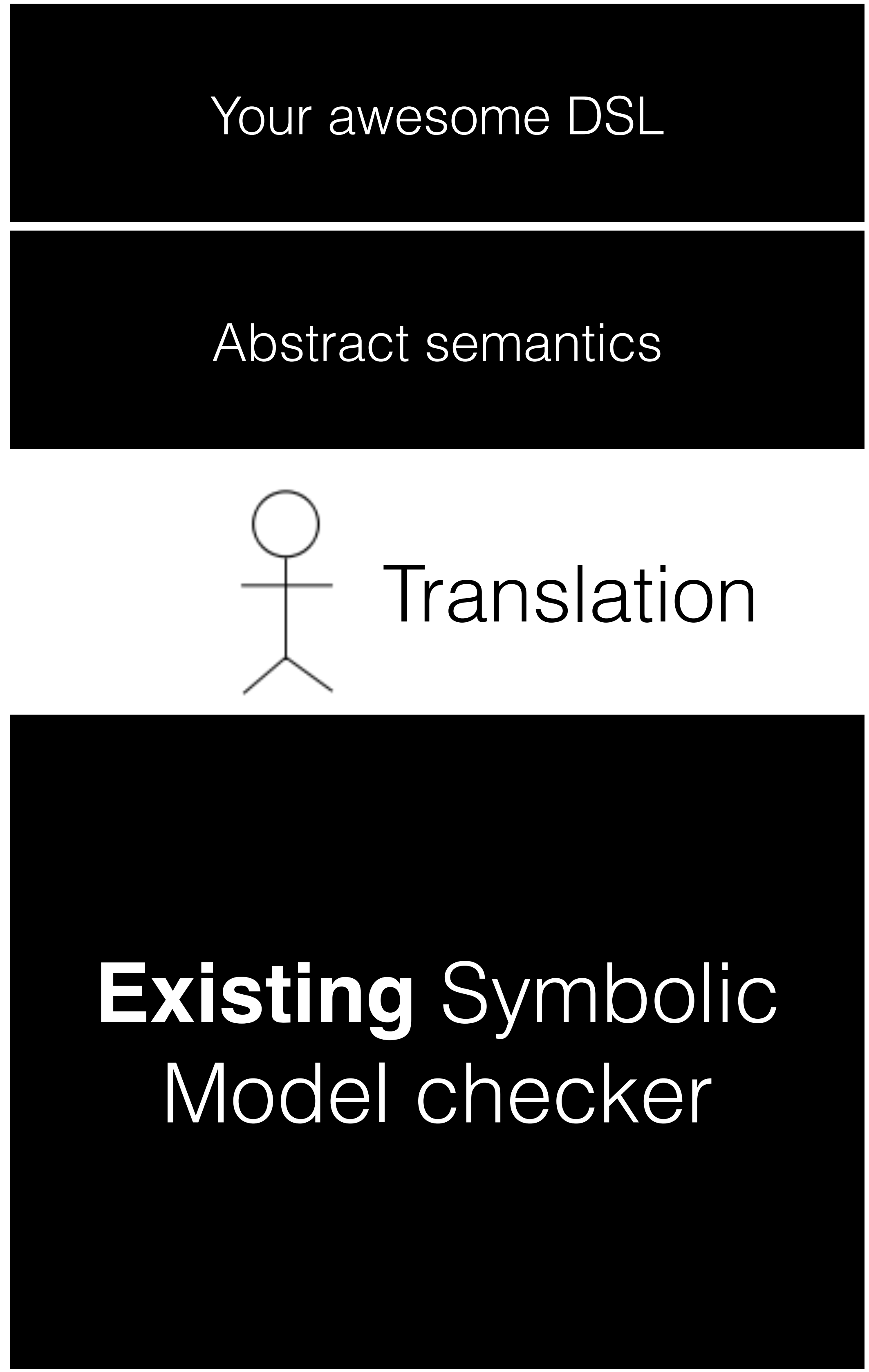
Your awesome DSL

Abstract semantics



Translation

Existing Symbolic
Model checker


Your awesome DSL
Abstract semantics
Translation
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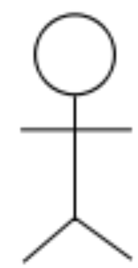
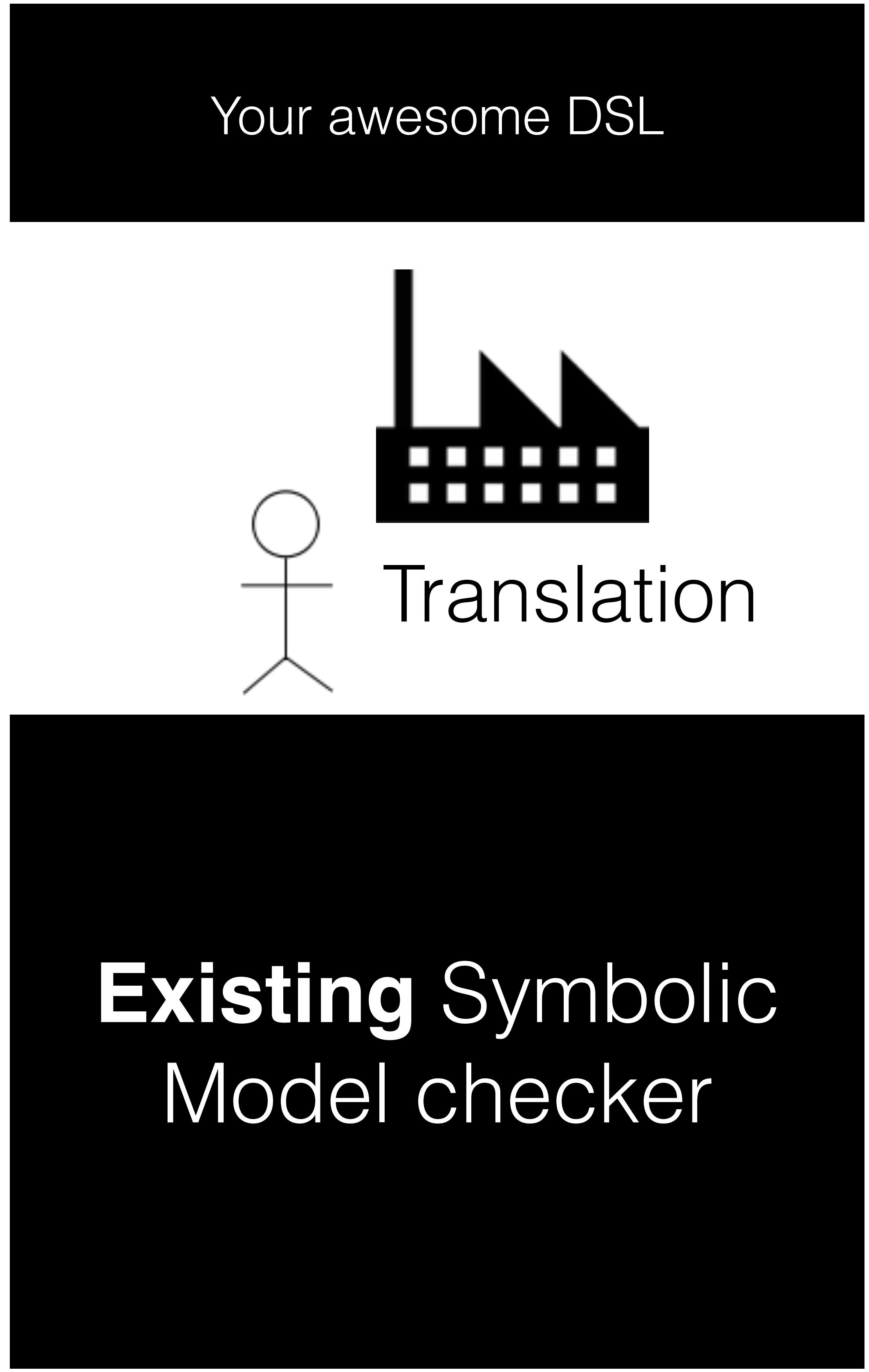
Your awesome DSL



Translation

Existing Symbolic Model checker


Your awesome DSL
Translation
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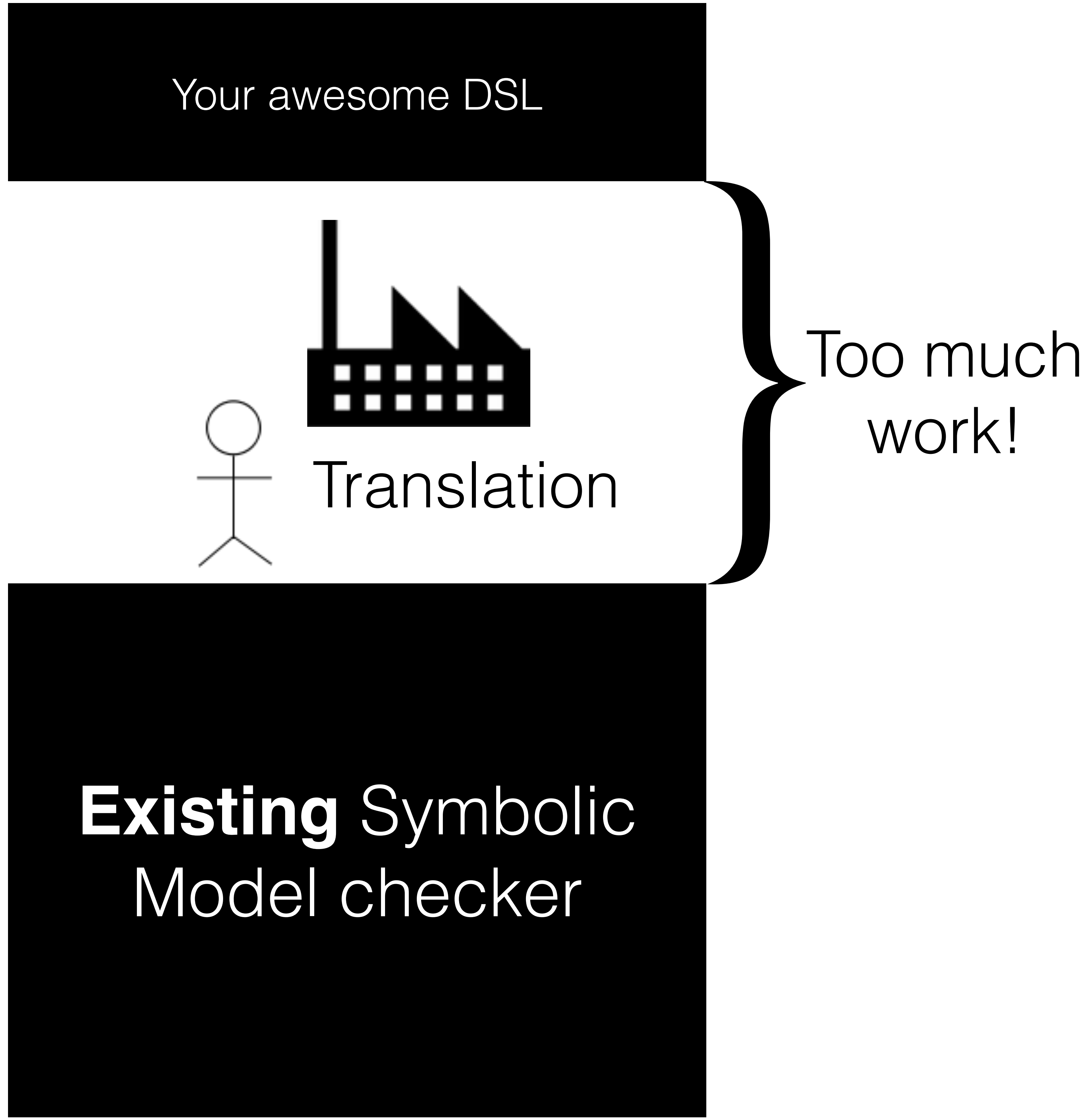
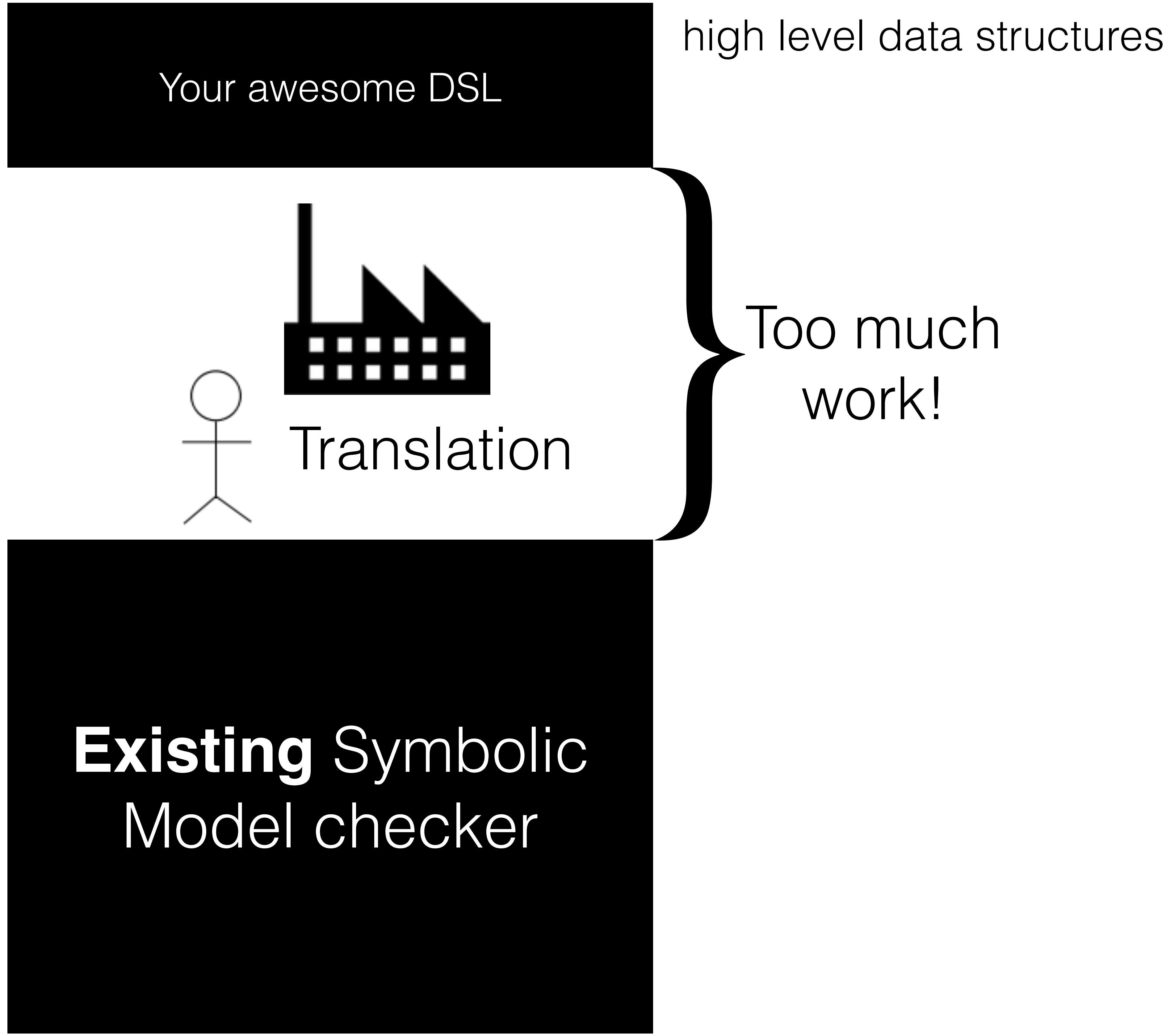

Your awesome DSL
Translation
Too much work!
Existing Symbolic Model checker


high level data structures
Your awesome DSL
Translation
Too much work!
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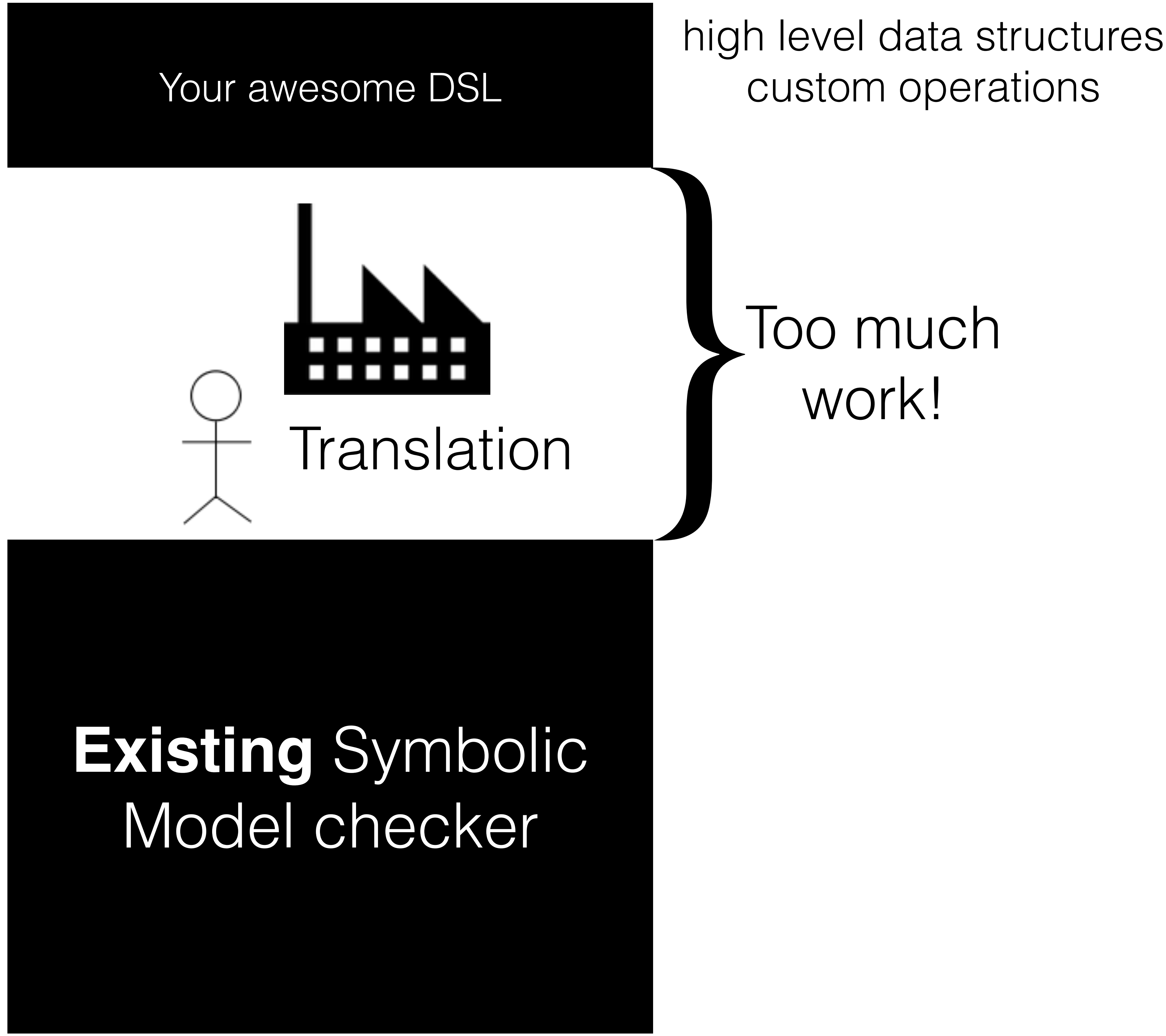
Your awesome DSL
high level data structures
custom operations
Translation
Too much
work!
Existing Symbolic
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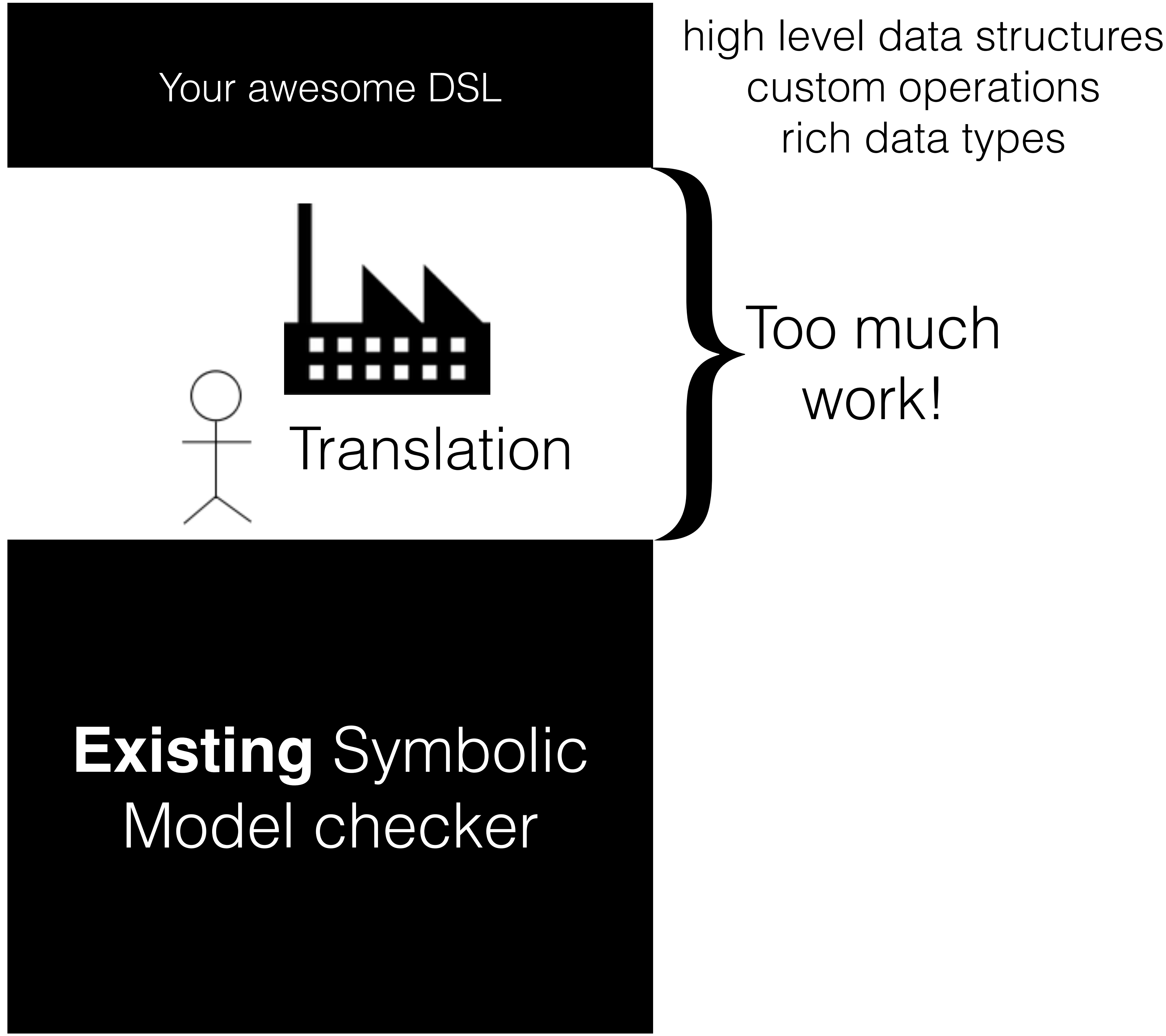
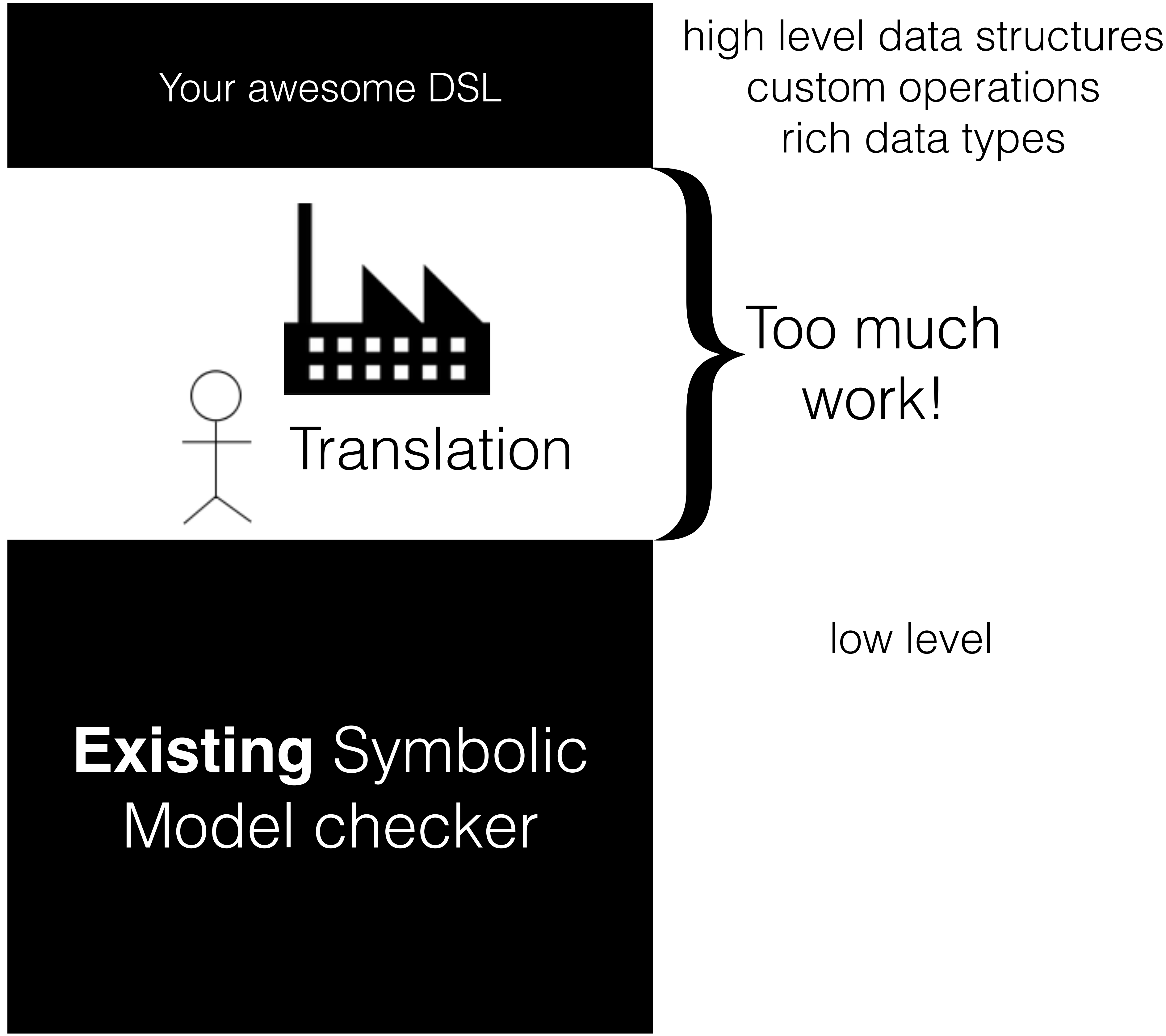

Your awesome DSL
high level data structures
custom operations
rich data types
Translation
Too much work!
Existing Symbolic Model checker


Your awesome DSL
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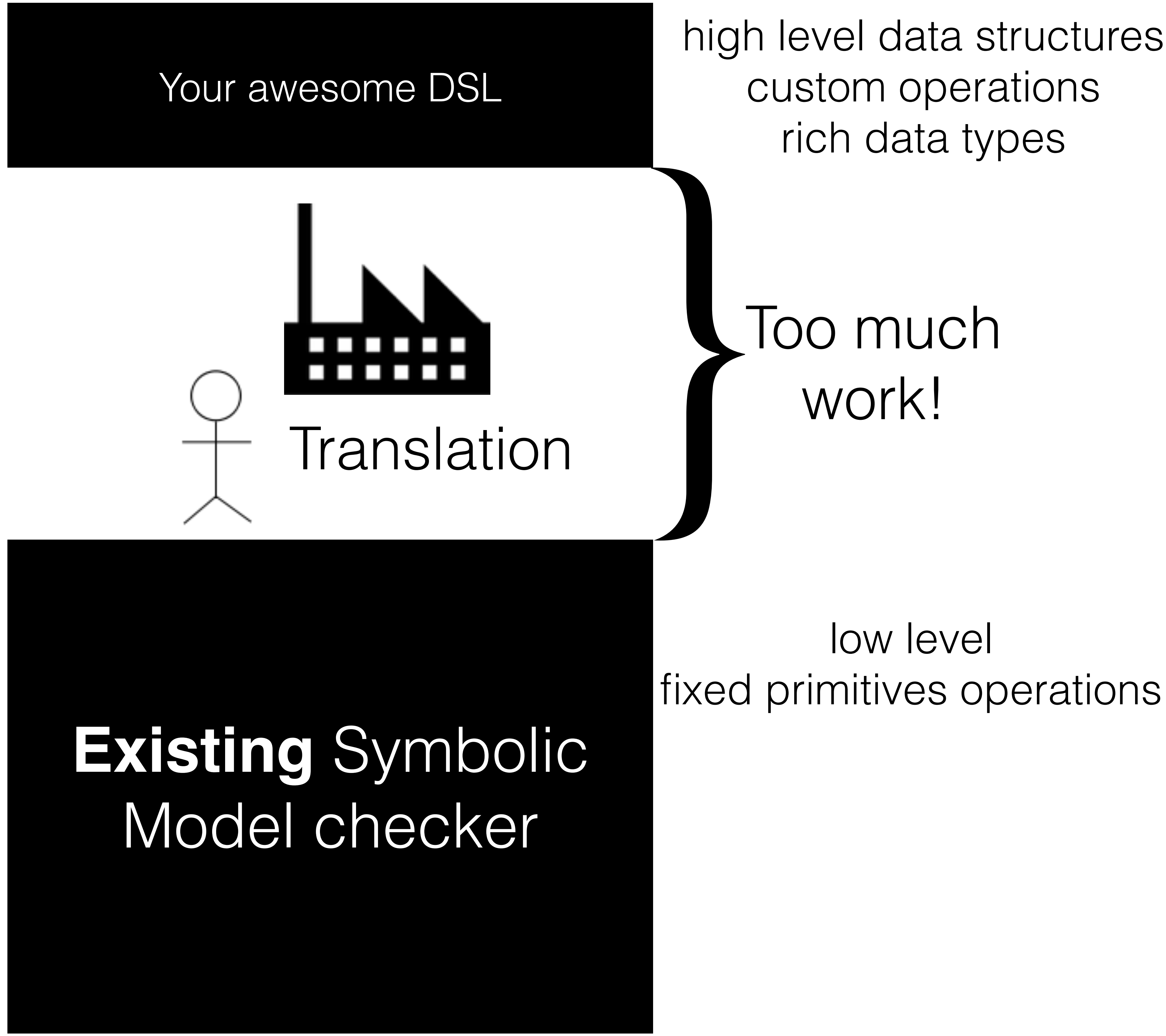
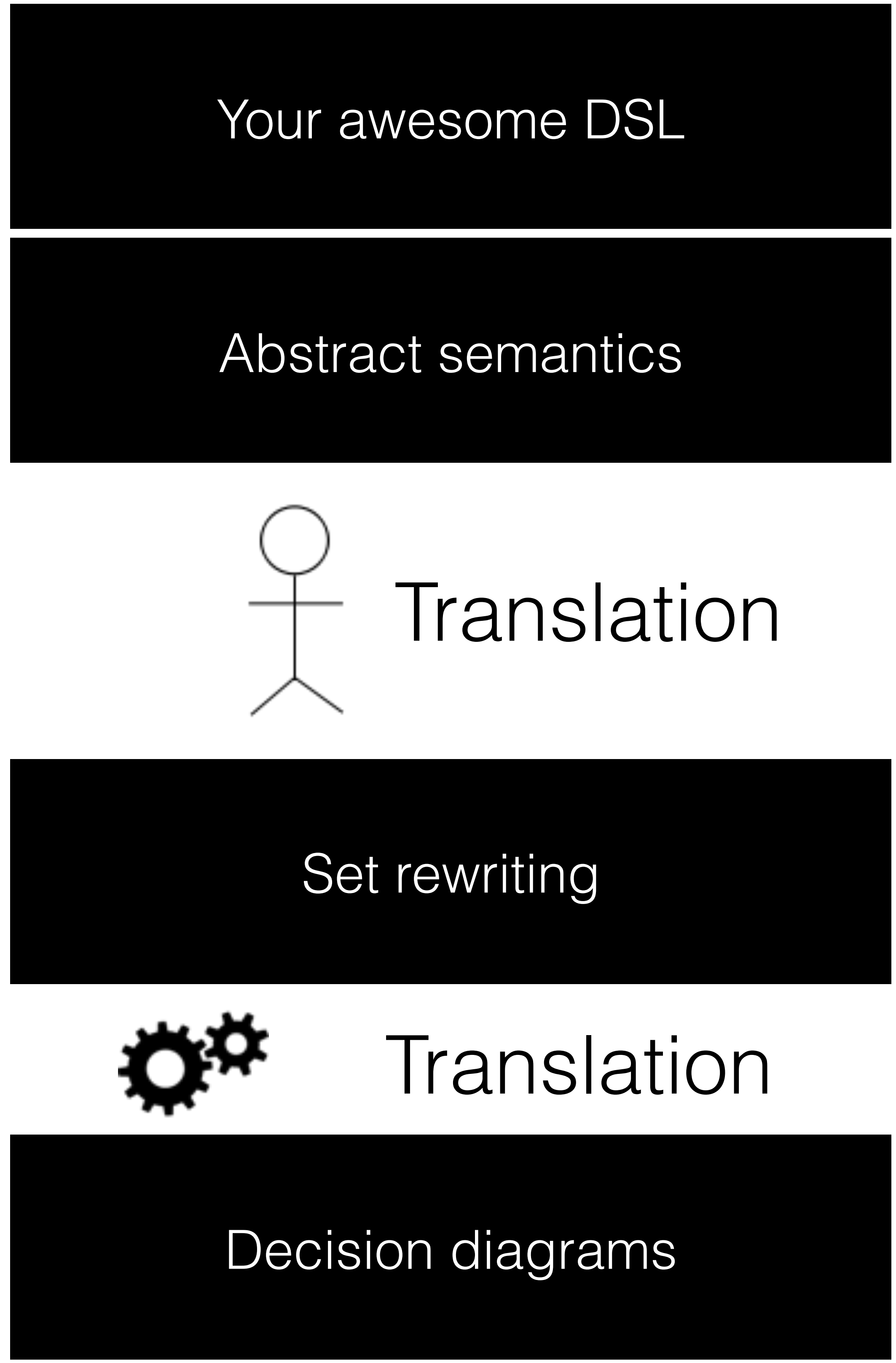
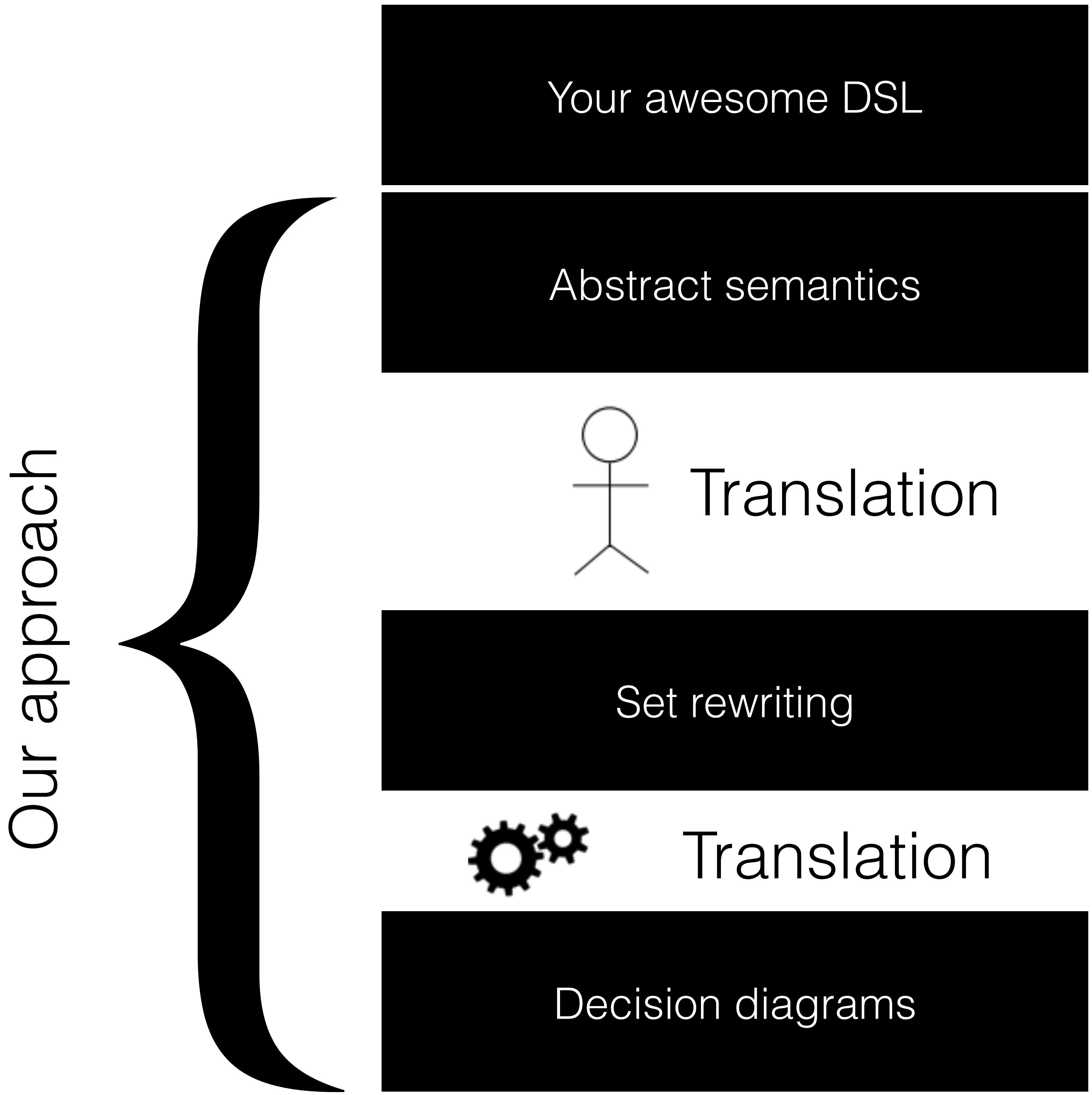
Translation
Too much
work!
low level
Existing Symbolic
Model checker


Your awesome DSL
high level data structures
custom operations
rich data types
Translation
Too much
work!
Existing Symbolic
Model checker
low level
fixed primitives operations


Your awesome DSL
Abstract semantics
Translation
Set rewriting
Translation
Decision diagrams


Your awesome DSL
Abstract semantics
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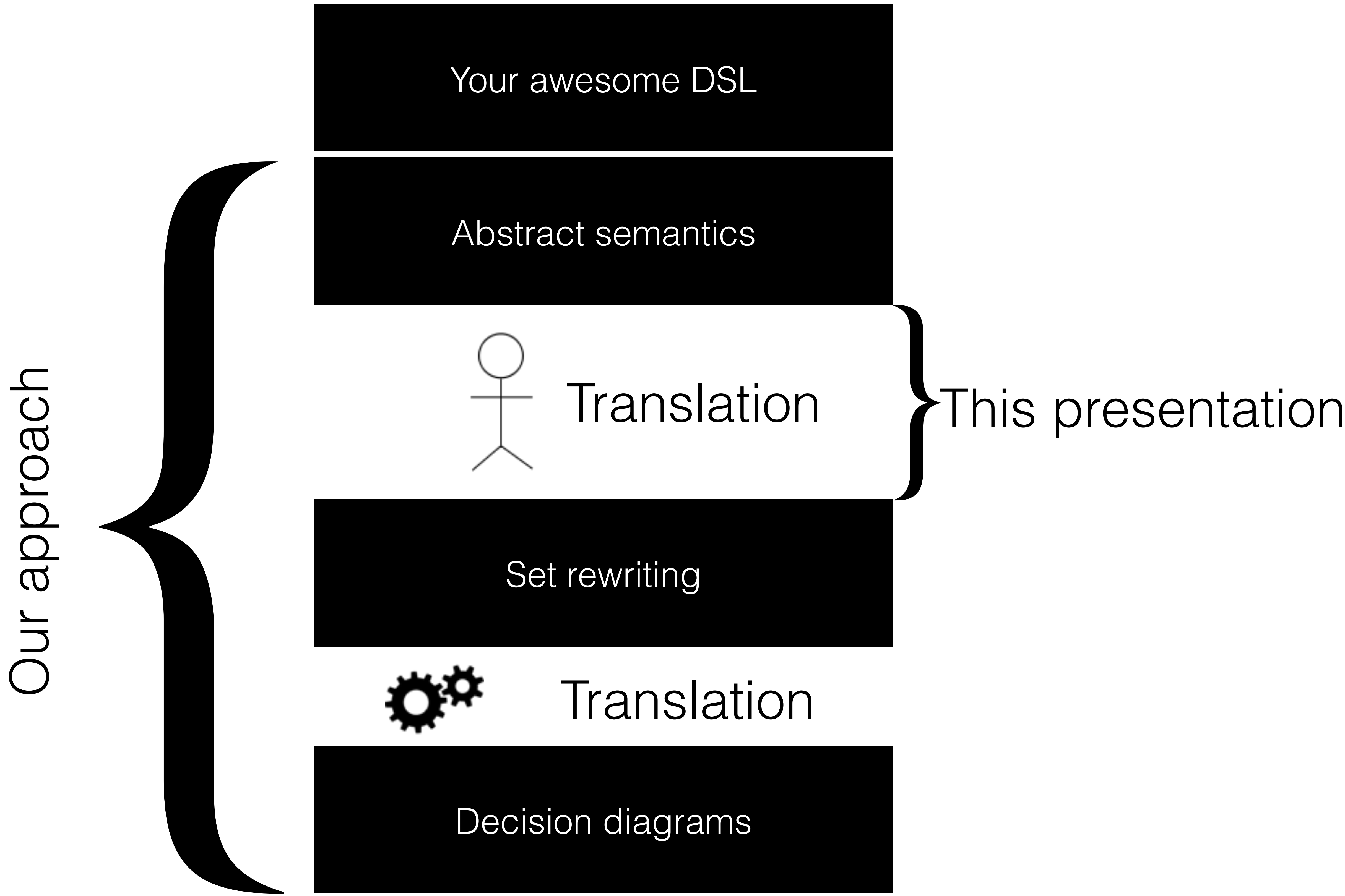
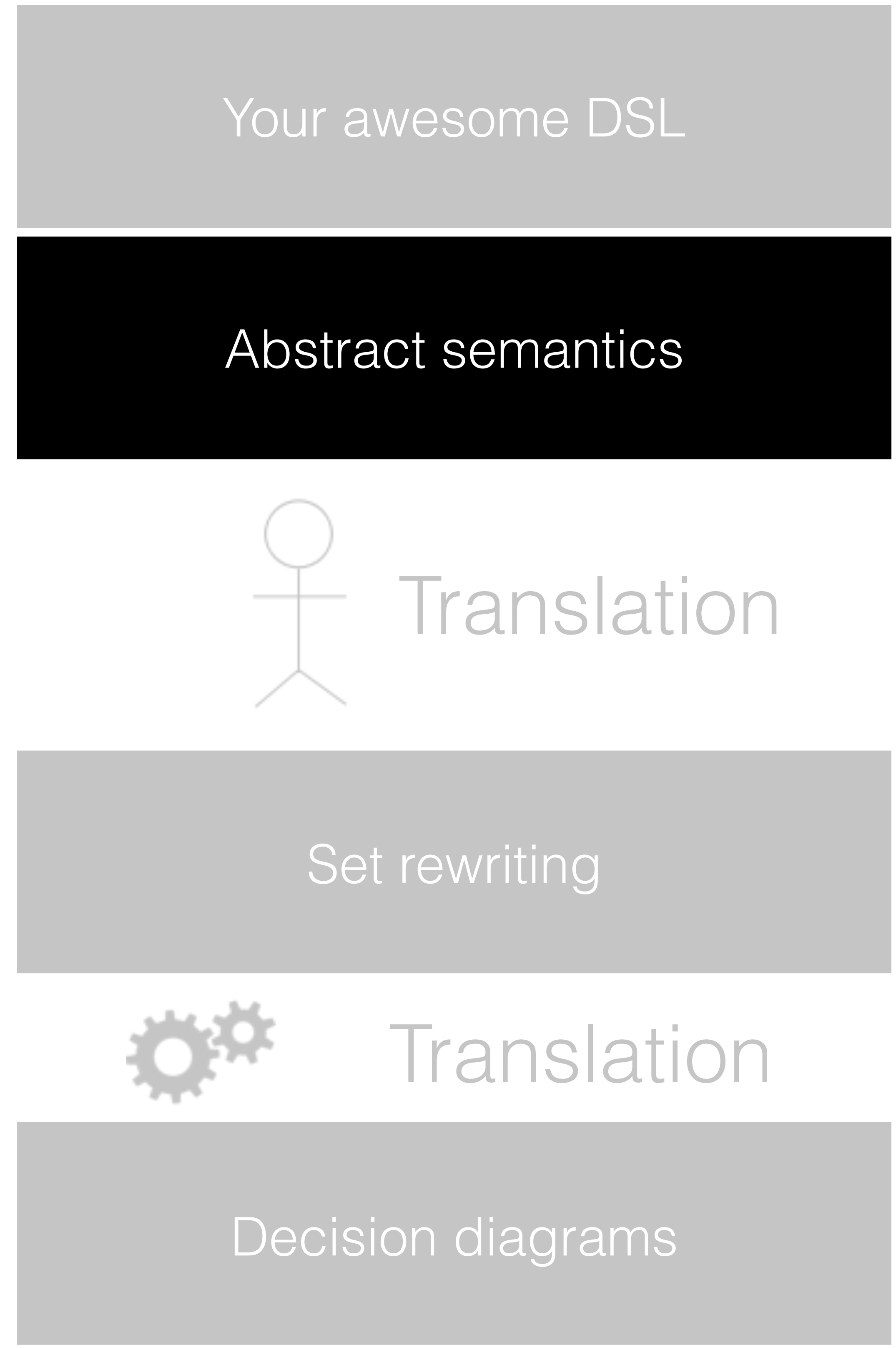
Translation
Decision diagrams
Our approach


Your awesome DSL
Abstract semantics
Translation
This presentation
Set rewriting
Translation
Decision diagrams
Our approach

Abstract semantics

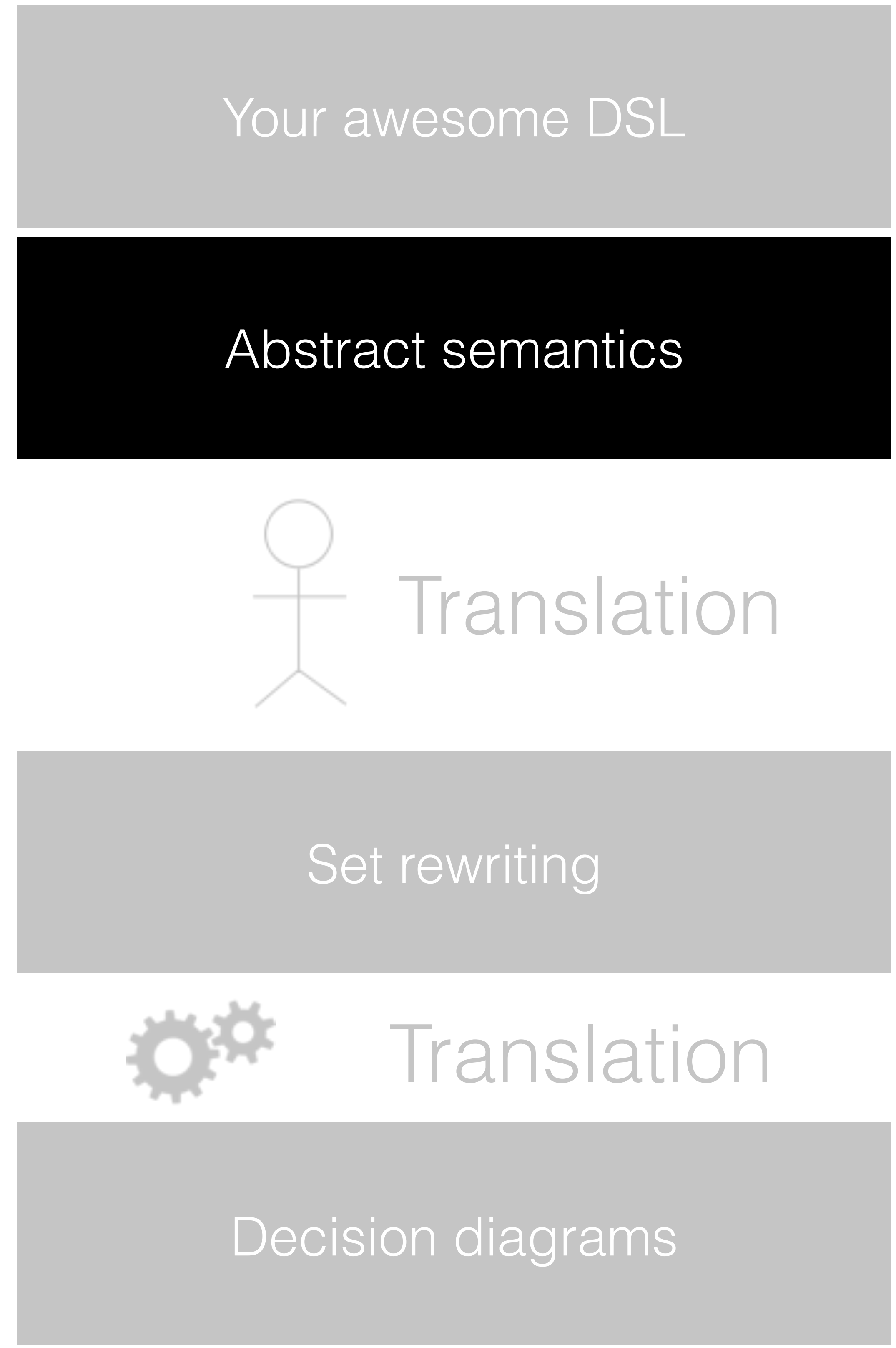
In context



Abstract semantics

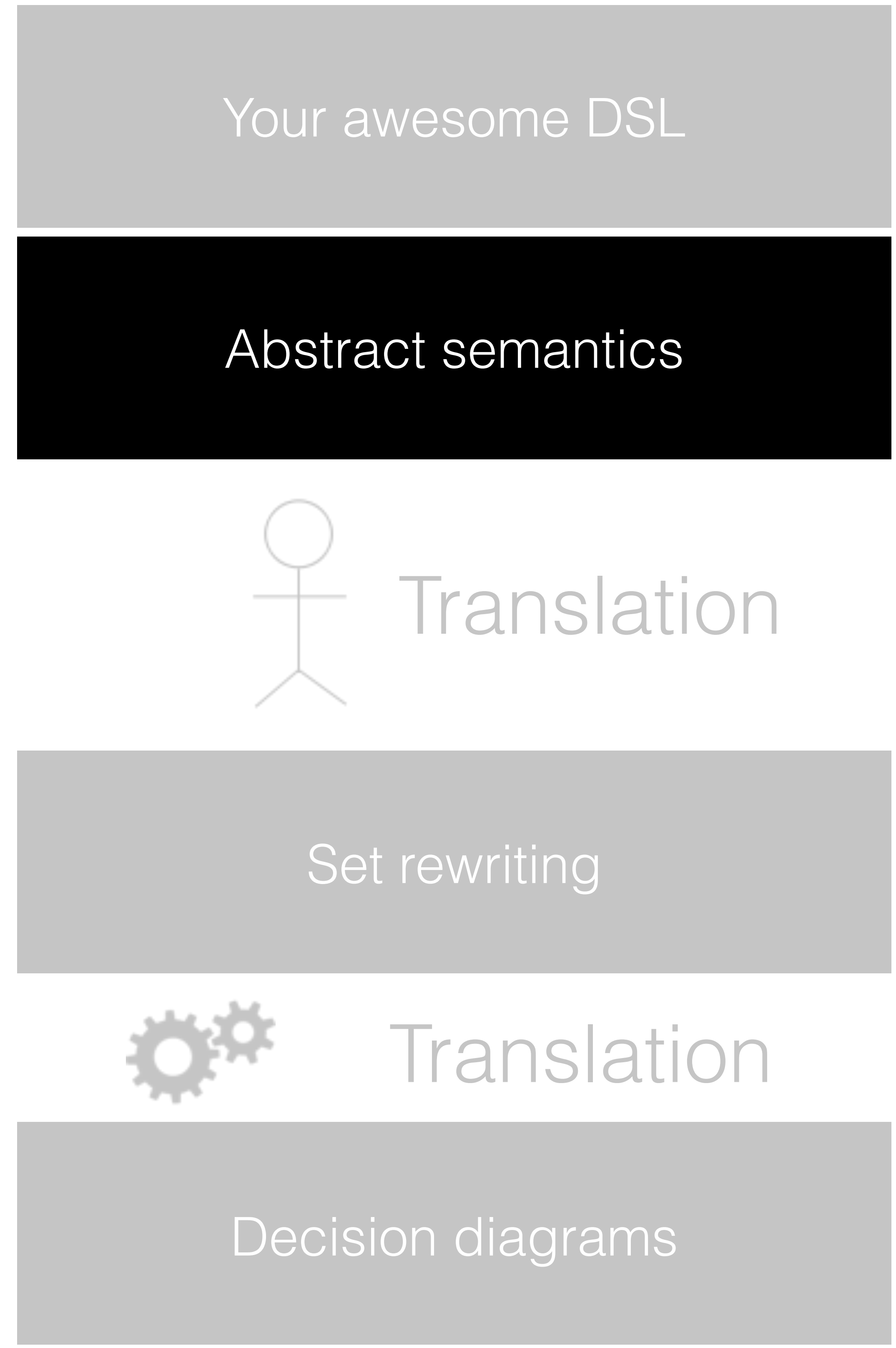
In context

- High level representation



Abstract semantics

In context



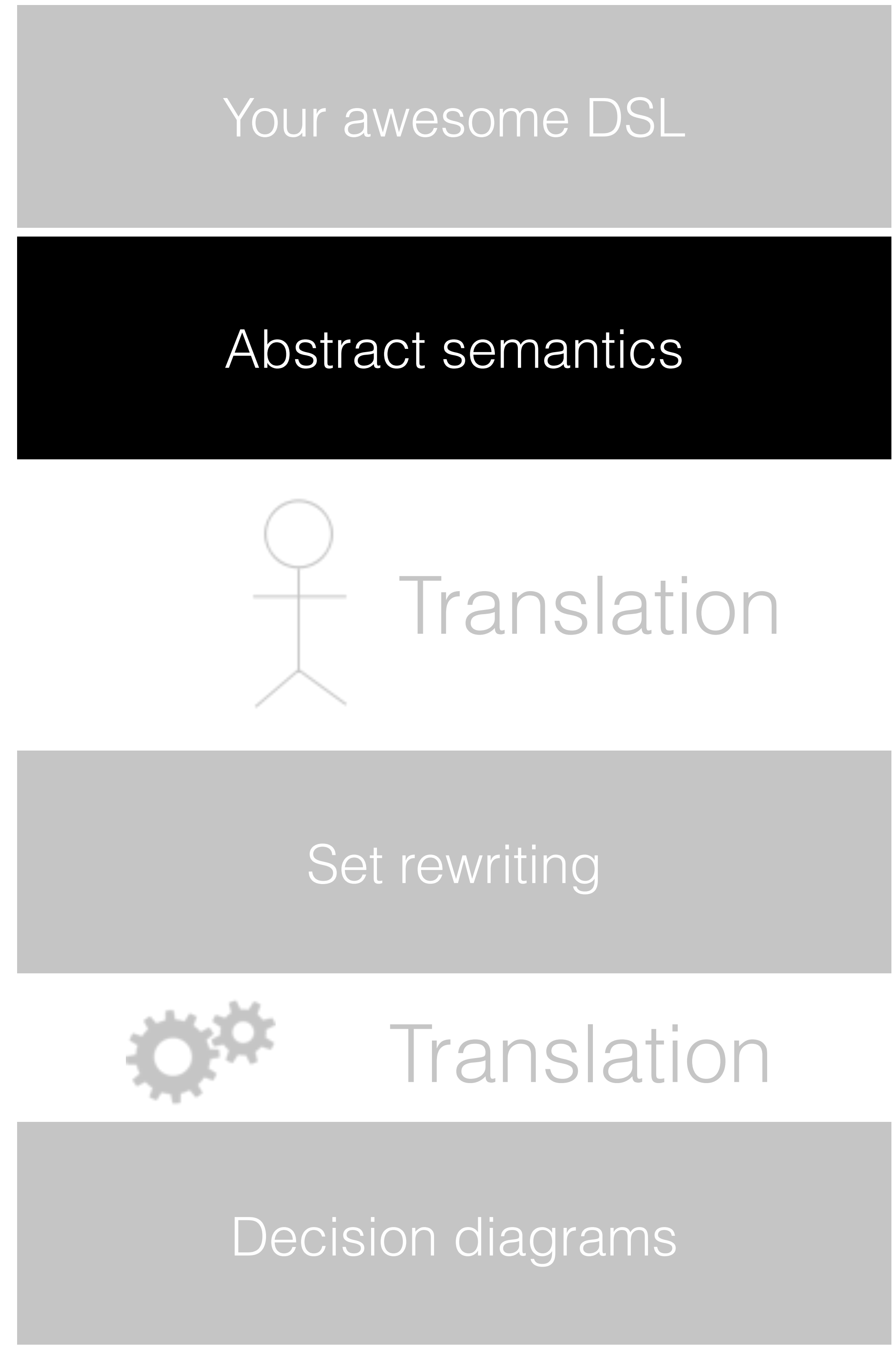
- High level representation
- Suitable for humans

Abstract semantics

In context

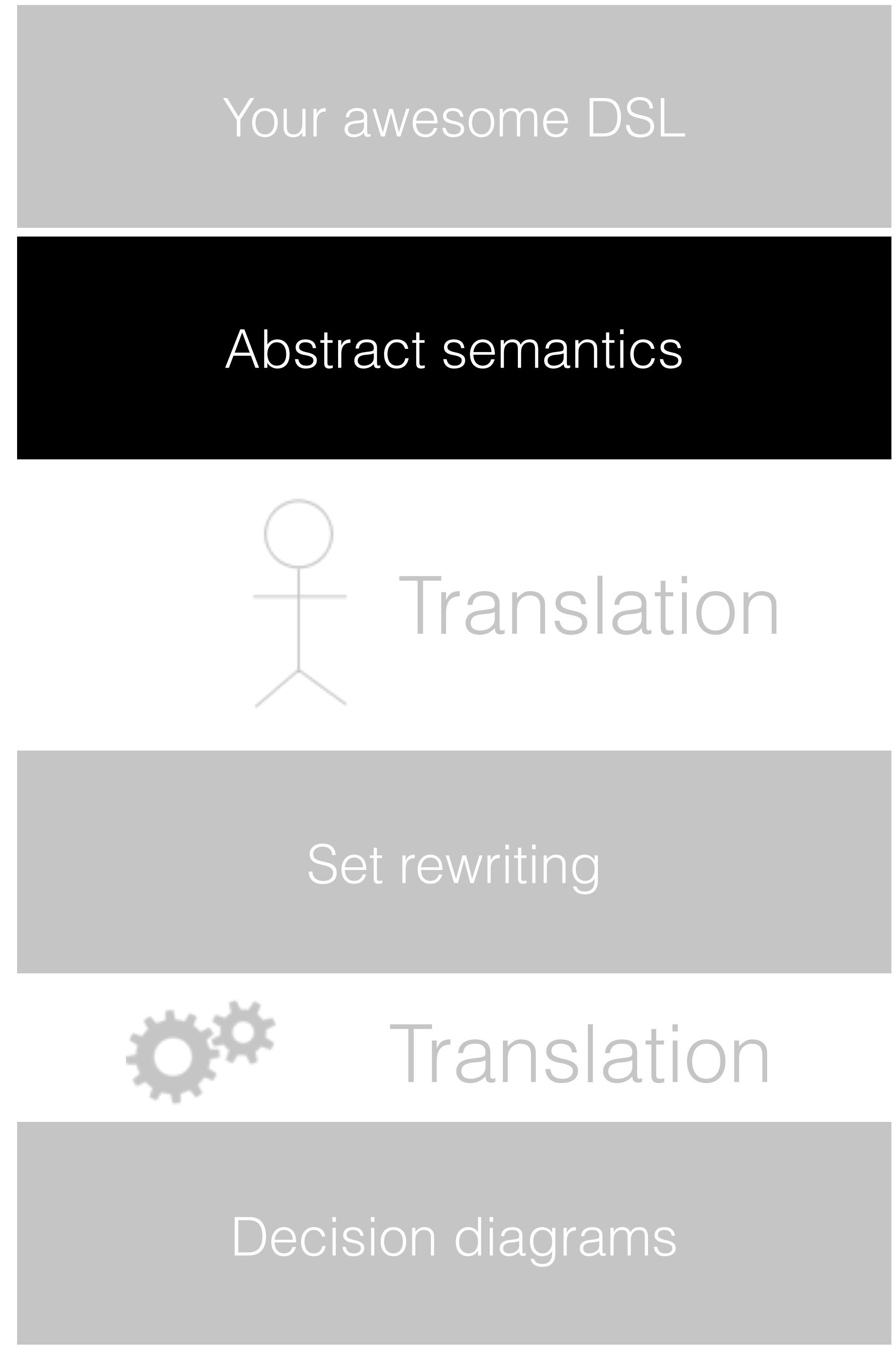
- We use Simple SOS rules:

$$\frac{c(s)}{s \to a(s)}$$



Abstract semantics

In context



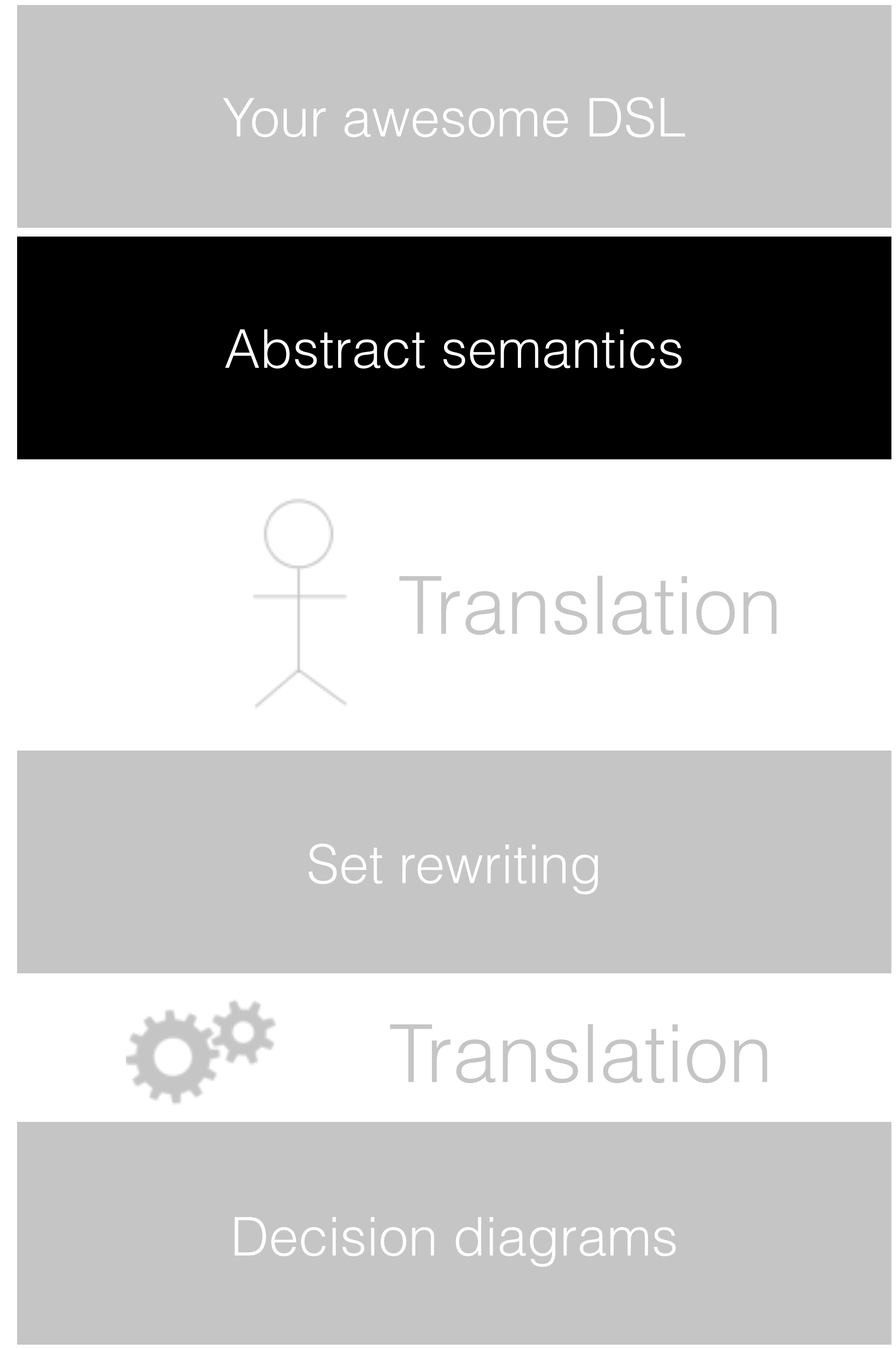
- We use Simple SOS rules:

$$\frac{c(s)}{s \rightarrow a(s)}$$

- s ∈ States

Abstract semantics

In context



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Abstract semantics

In context

- We use Simple SOS rules:

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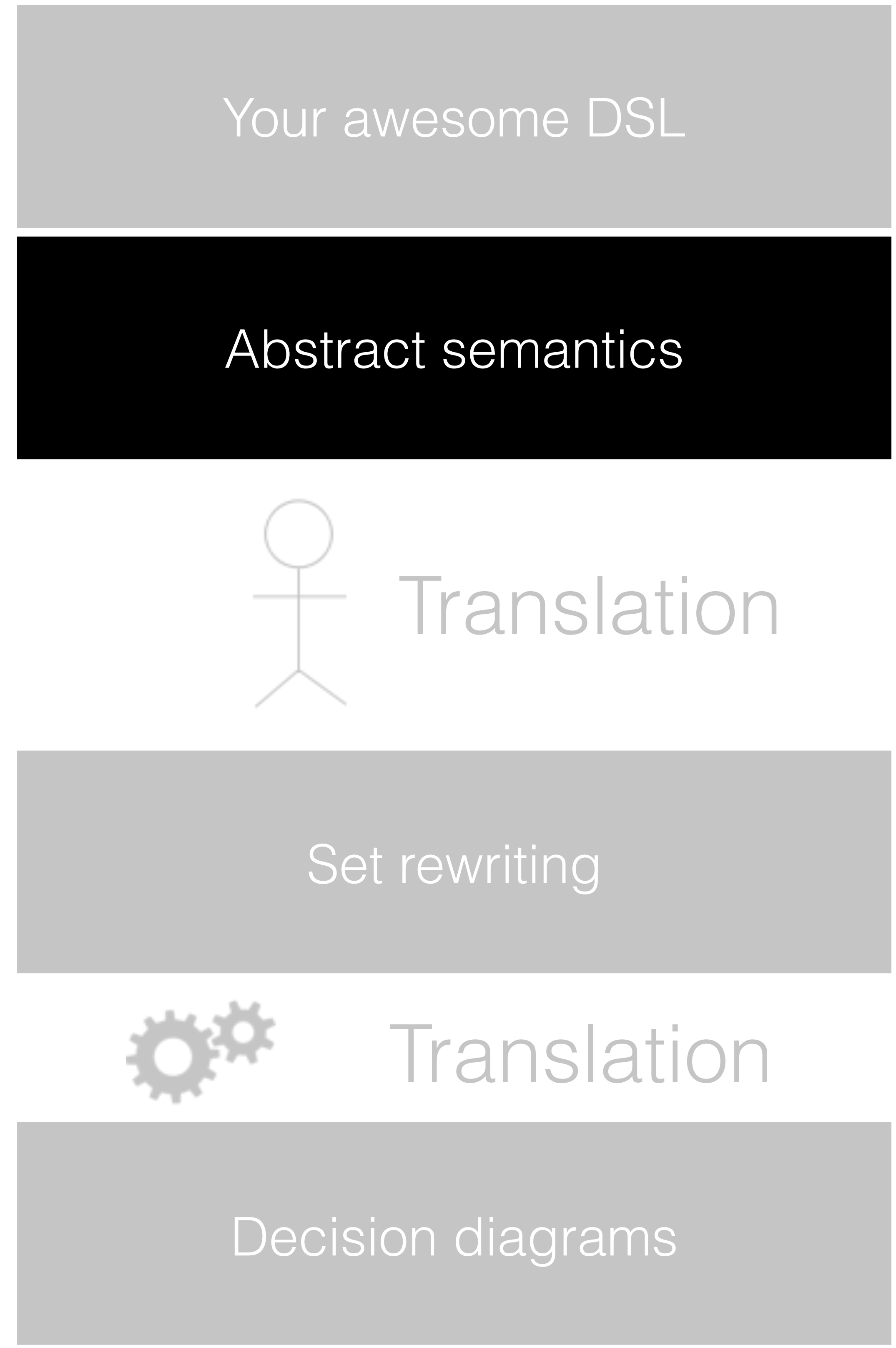
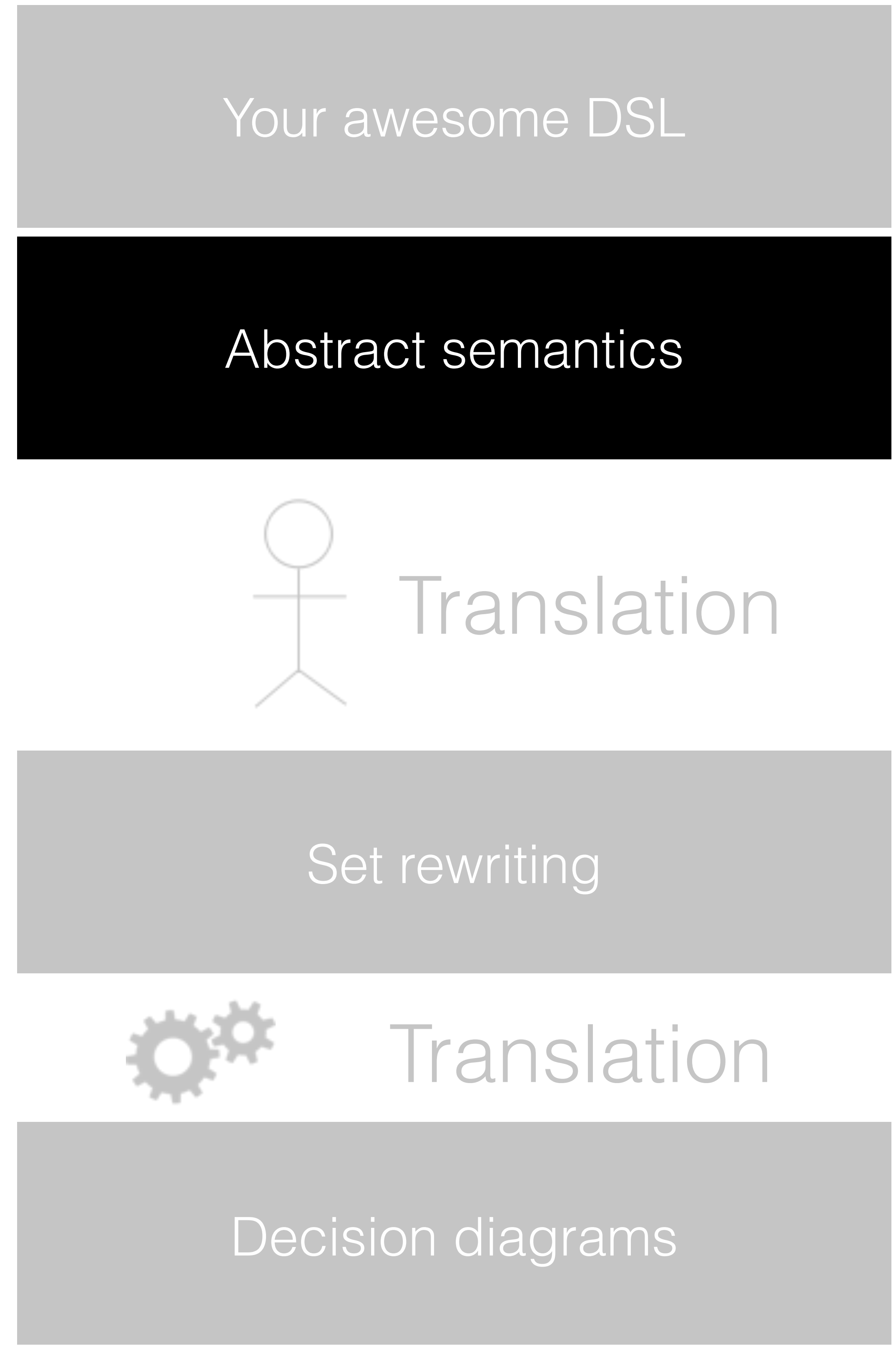
- c: States ⟶ $\mathbb{B}$, c ∈ Cond
- a: States ⟶ States, a ∈ Act



Abstract semantics

In context

$$\frac{c(s)}{s \rightarrow a(s)}$$

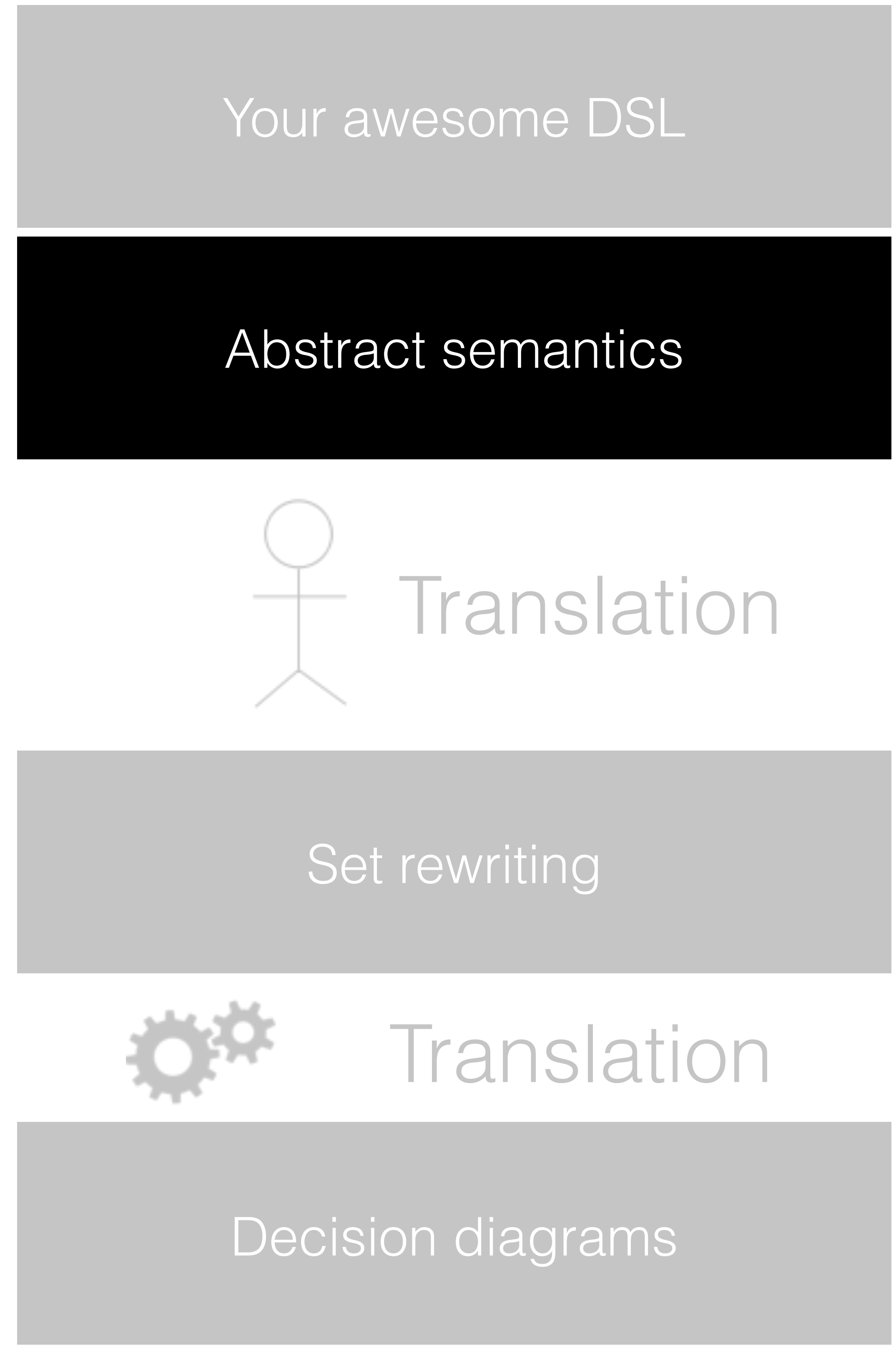


Abstract semantics

In context

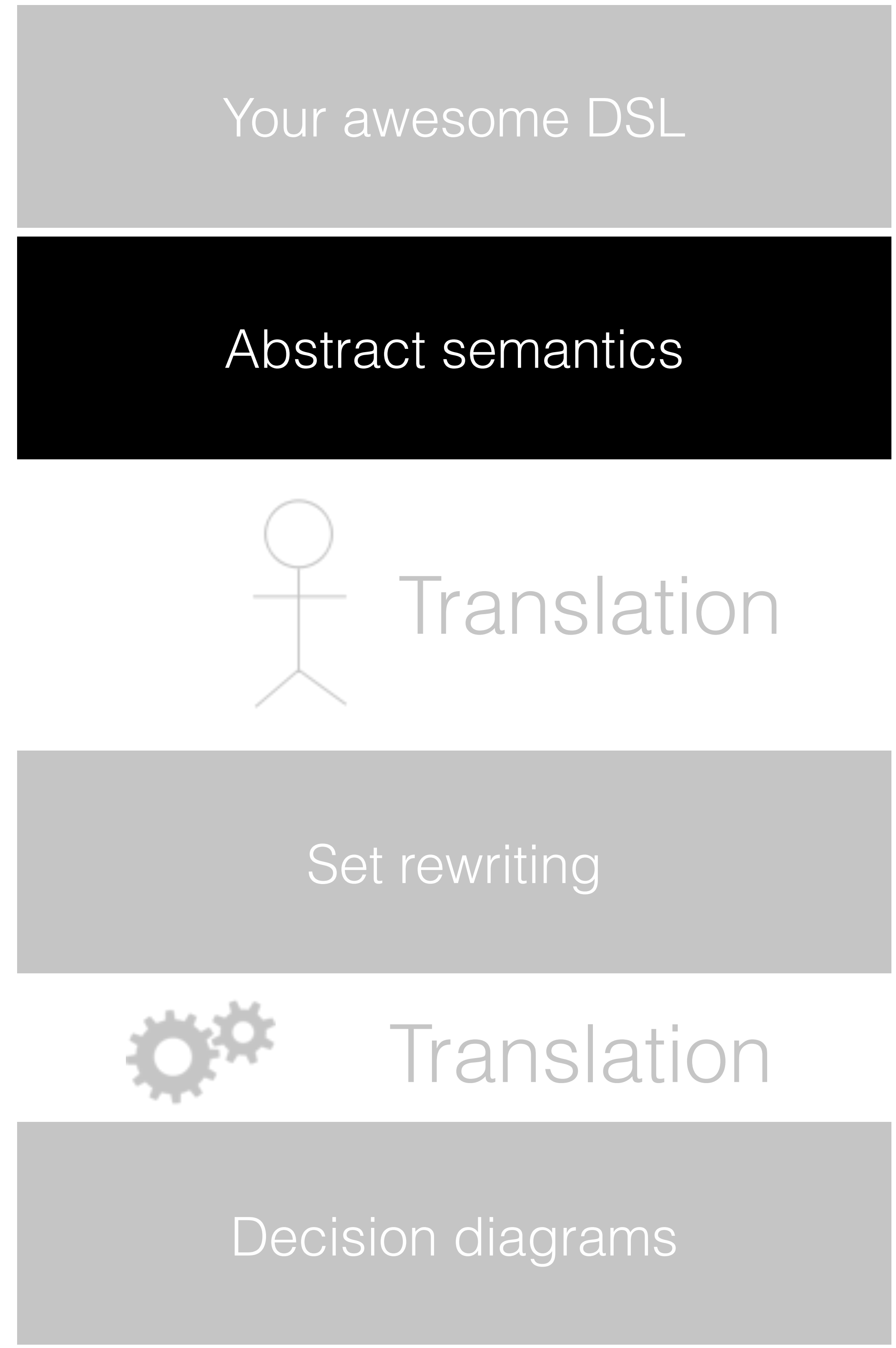
$$\frac{c(s)}{s \to a(s)}$$

- $c_1 \wedge c_2 \in \mathrm{Cond}$



Abstract semantics

In context



$$\frac{c(s)}{s \to a(s)}$$

- $c_1 \wedge c_2 \in \text{Cond}$
- $c \circ a \in \text{Cond}$

Abstract semantics

In context

$$\frac{c(s)}{s \to a(s)}$$

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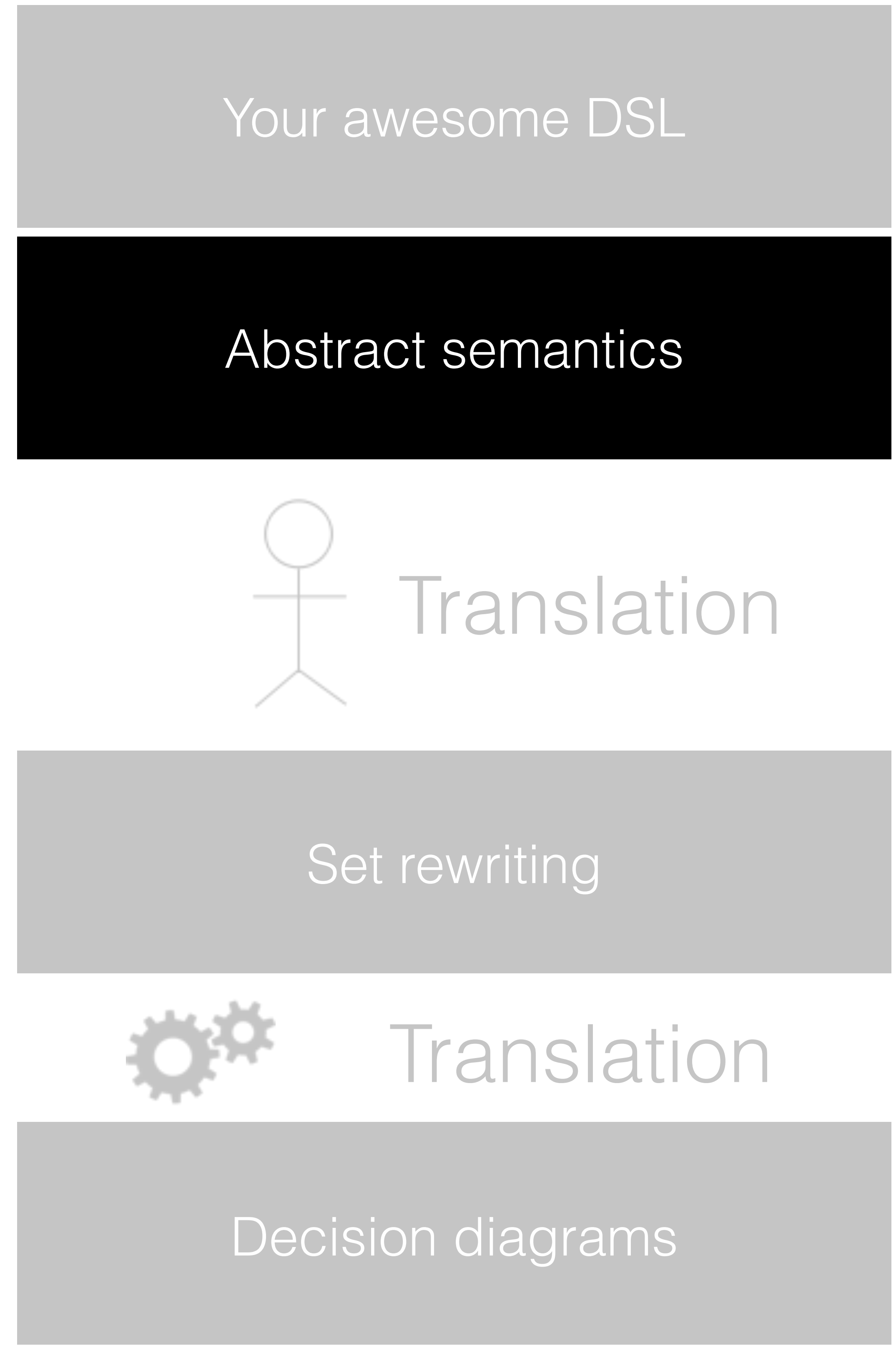
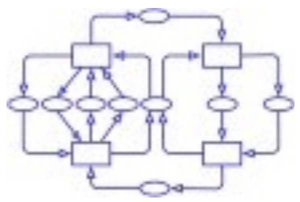
- $c \circ a \in$ Cond
- $a \circ a \in$ Act

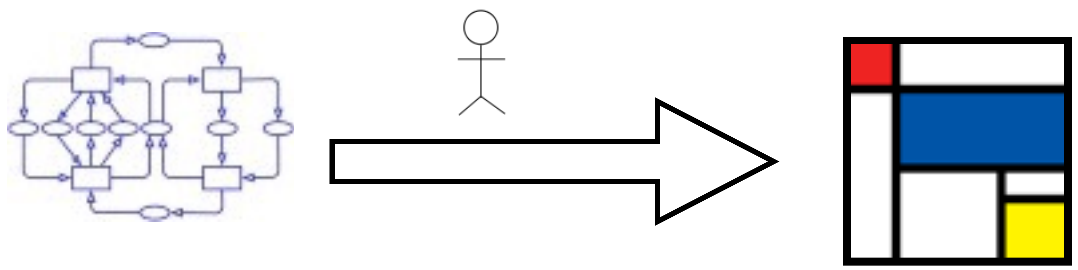


Goal

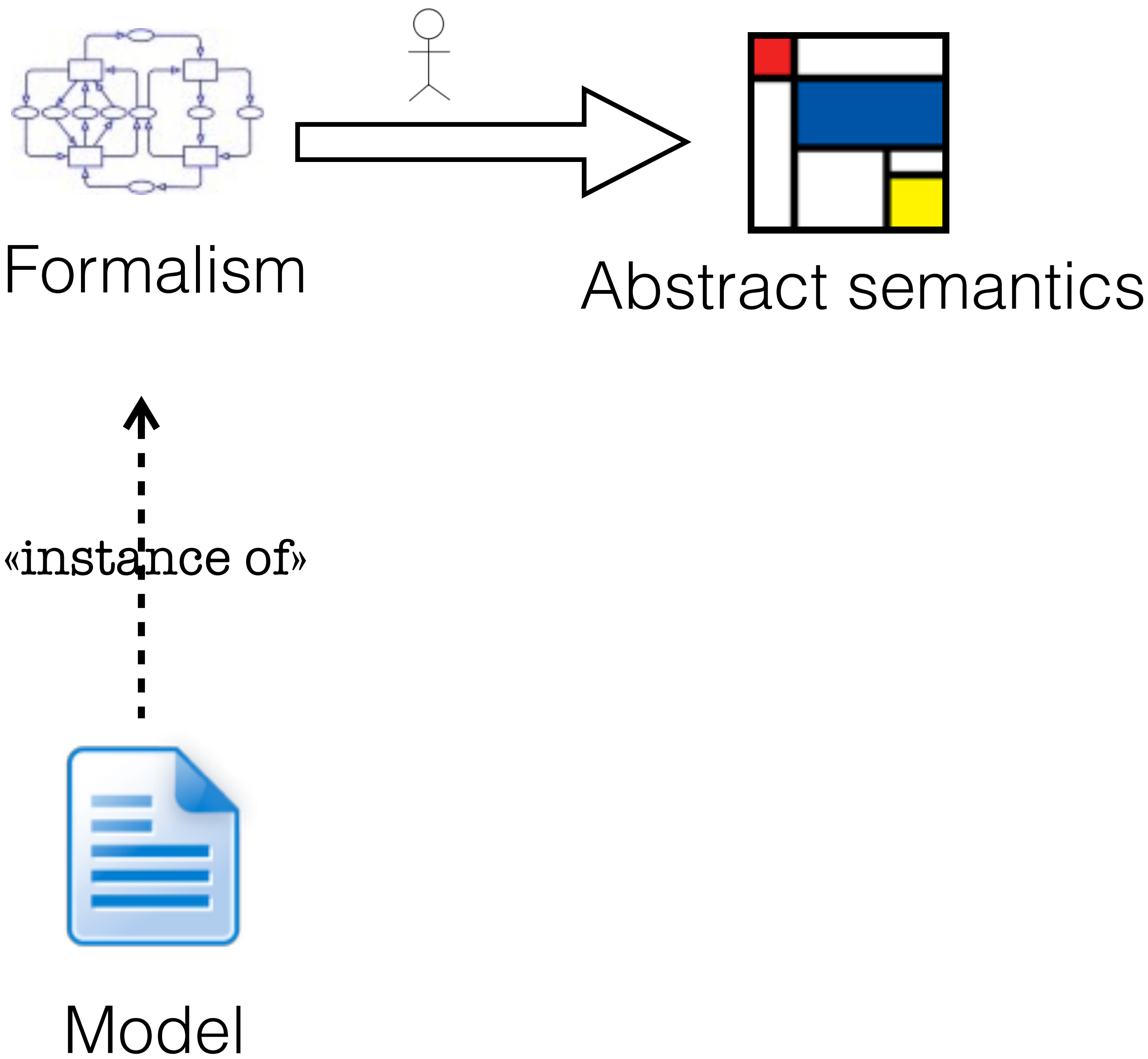


Formalism

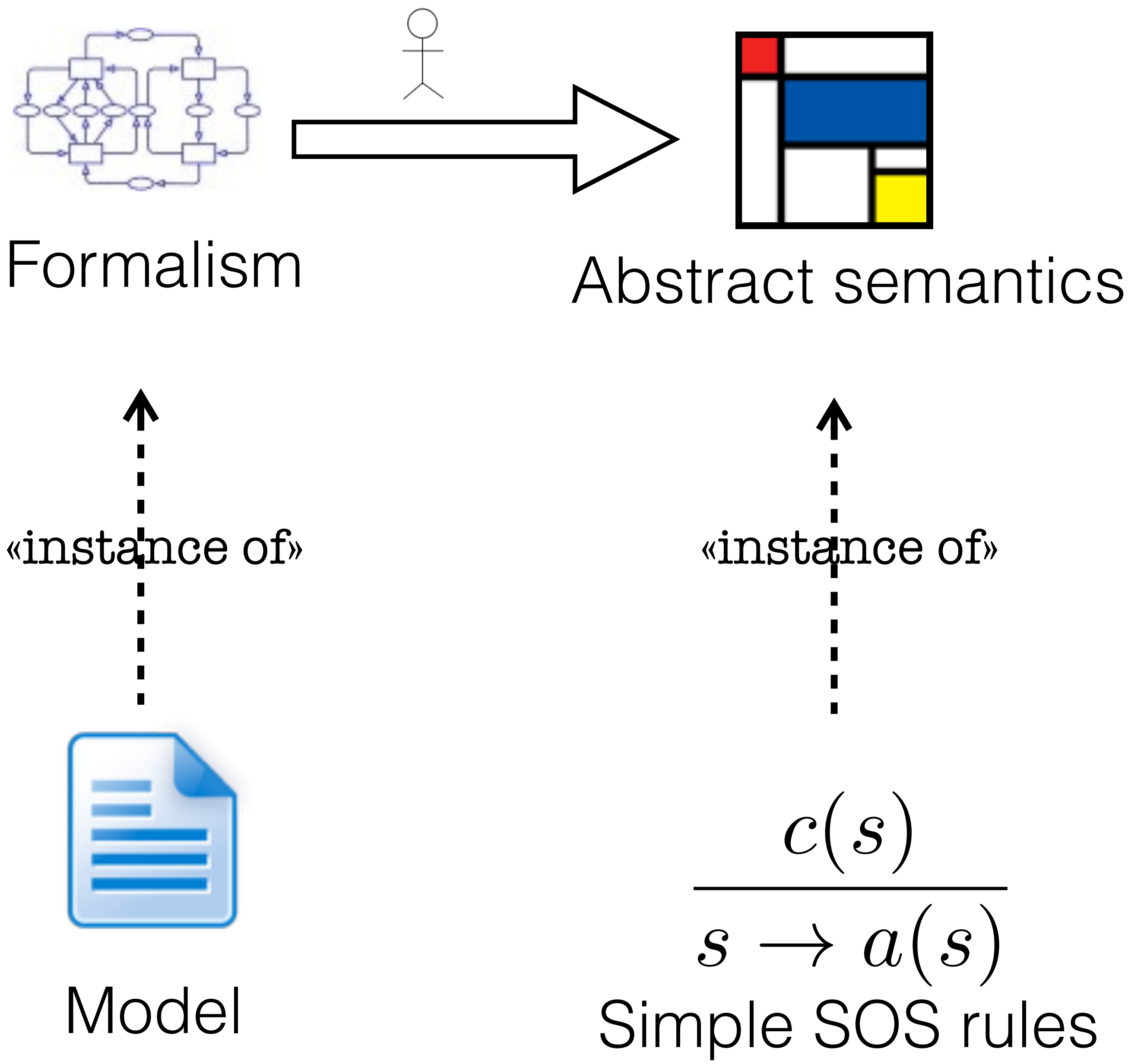
Goal



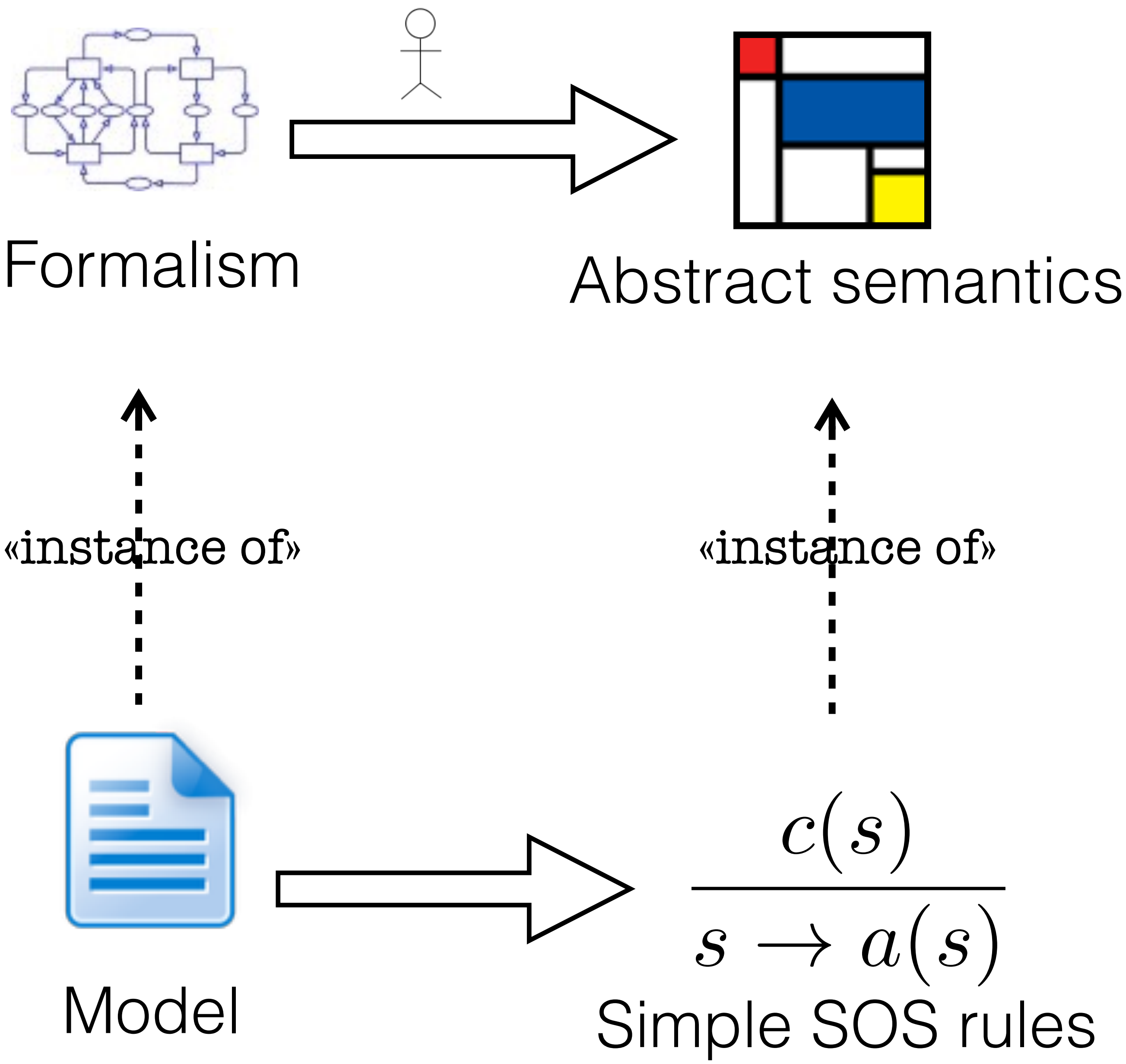
Goal



Goal


Formalism
Abstract semantics
«instance of»
«instance of»
Model
c(s)
s → a(s)
Simple SOS rules

Goal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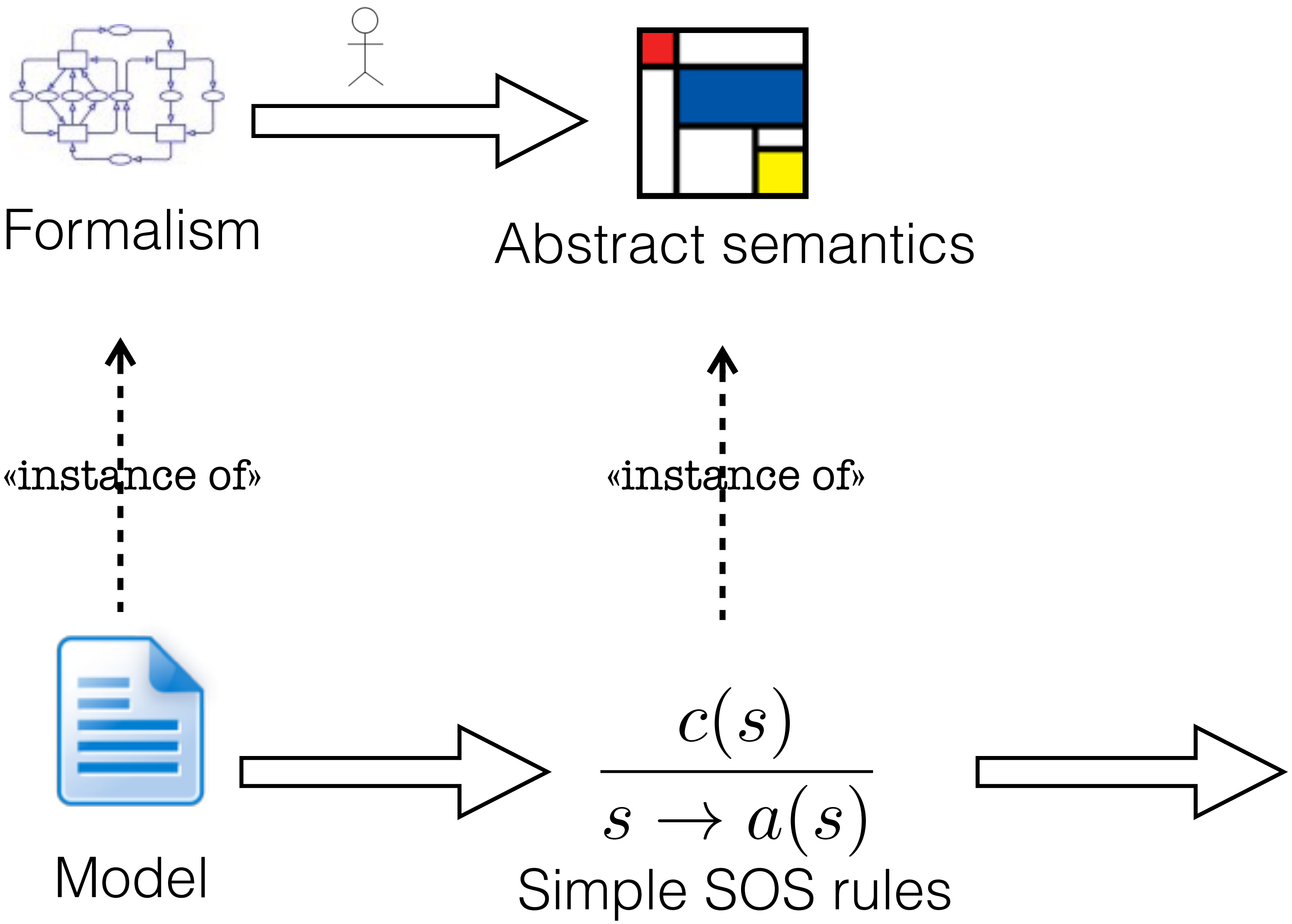
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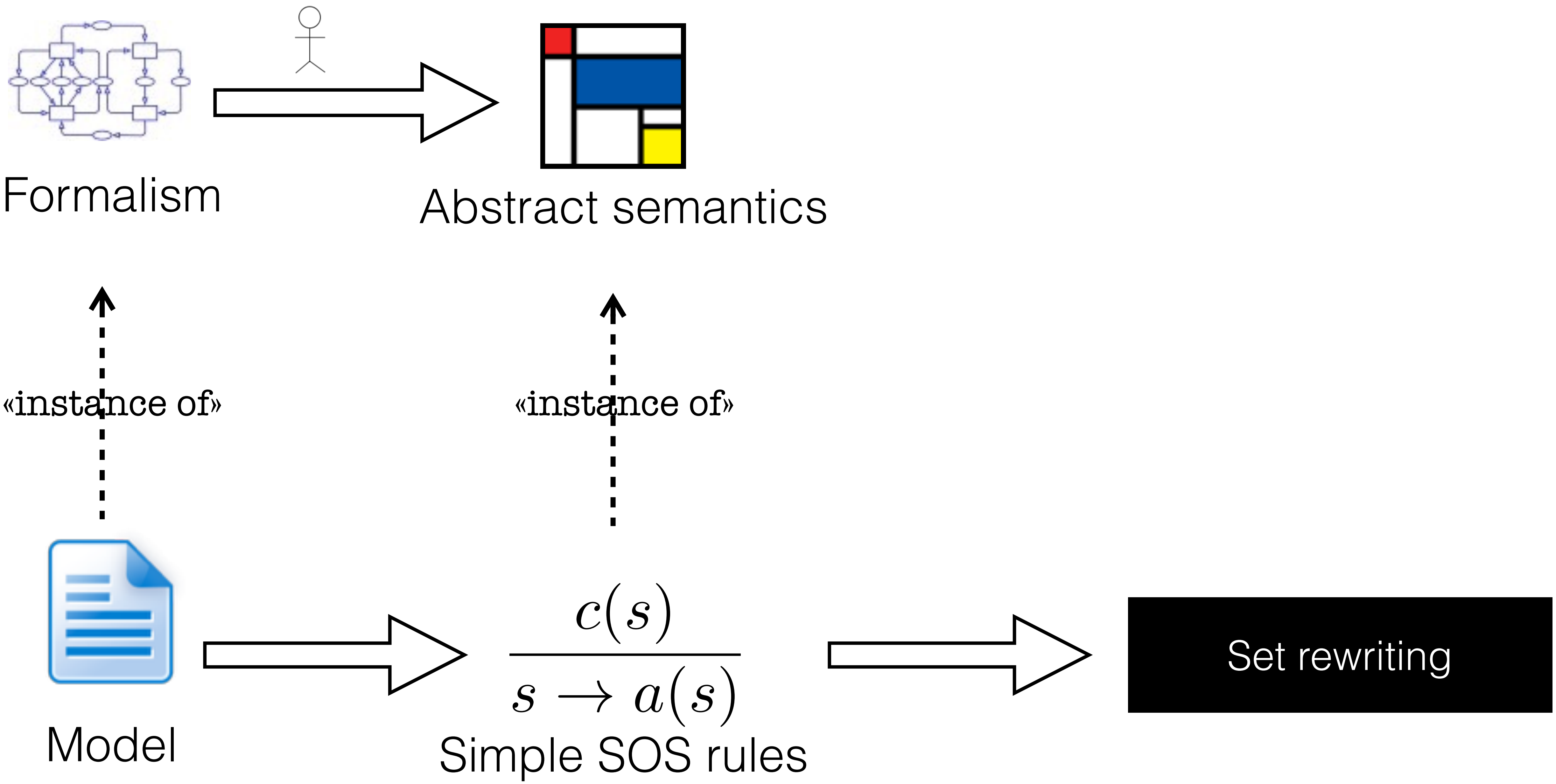
Formalism
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«instance of»
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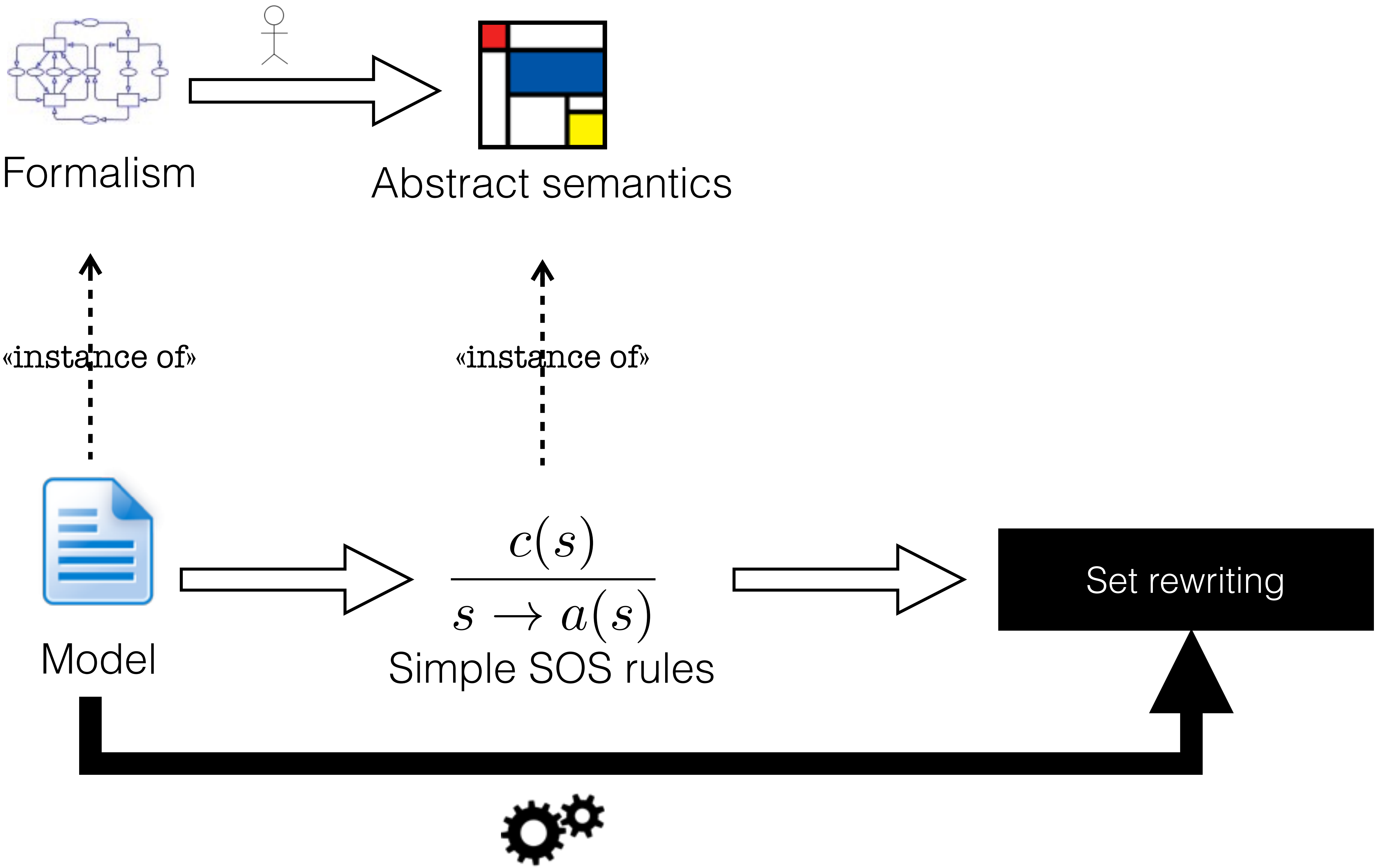
Model
$\frac{c(s)}{s \to a(s)}$
Simple SOS rules
Set rewriting

Goal


Formalism
Abstract semantics
«instance of»
«instance of»
Model
$\frac{c(s)}{s \rightarrow a(s)}$
Simple SOS rules
Set rewriting

The translation

$$\frac{c(s)}{s \to a(s)}$$

⇒

Set rewriting

The translation

$$\frac{c(s)}{s \to a(s)} \quad \Longrightarrow \quad \text{Set rewriting}$$

- stateComp: State $\longrightarrow T_{\Sigma}$

The translation

$$\frac{c(s)}{s \to a(s)} \quad \Longrightarrow \quad \text{Set rewriting}$$

- stateComp: State $\longrightarrow T_{\Sigma}$
- comp: SimpleSOS $\longrightarrow$ Strategies

General principles

$$\frac{c(s)}{s \to a(s)} \quad \Longrightarrow \quad \text{Sequence(comp(c), comp(a))}$$

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General principles

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- $\text{comp}(c_1 \wedge c_2) = \text{Sequence}(\text{comp}(c_1), \text{comp}(c_2))$
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General principles

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- comp(a) = userActComp(a)

General principles

$$\frac{c(s)}{s \to a(s)} \implies \text{Sequence(comp(c), comp(a))}$$

- If c is false, comp(c) fails
- comp($c_1 \wedge c_2$) = Sequence(comp(c_1), comp(c_2))
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- comp(a) = userActComp(a)
- comp(c) = userPredComp(c)

Case Study: Petri nets

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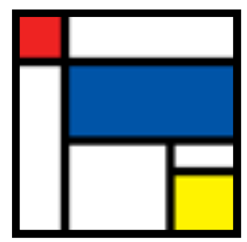

Abstract semantics

$$\frac{m \geq in(t)}{m \rightarrow_t m - in(t) + out(t)}$$

Case Study: Petri nets

Abstract semantics

$$\frac{m \geq in(t)}{m \rightarrow_t m - in(t) + out(t)}$$

$$\frac{\bigwedge_{p \in P} test_{p,in(t)(p)}(m) \quad P = \{p_1, \ldots, p_{n'}\}}{m \rightarrow dec_{p_1,in(t)(p_1)} \circ \cdots \circ dec_{p_{n'},in(t)(p_{n'})} \circ inc_{p_1,in(t)(p_1)} \circ \cdots \circ inc_{p_{n'},in(t)(p_{n'})}(m)}$$

Case Study: Petri nets

Abstract semantics

$$\frac{m \geq in(t)}{m \to_t m - in(t) + out(t)}$$

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- userPredComp($test_{p,in(t)}$) = ???

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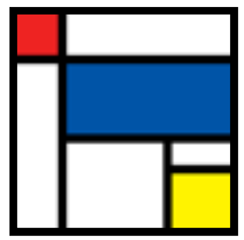


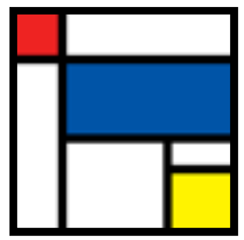
Abstract semantics

$$\frac{m \geq in(t)}{m \rightarrow_t m - in(t) + out(t)}$$

$$\frac{\bigwedge_{p \in P} test_{p,in(t)(p)}(m) \quad P = \{p_1, \ldots, p_{n'}\}}{m \rightarrow dec_{p_1,in(t)(p_1)} \circ \cdots \circ dec_{p_{n'},in(t)(p_{n'})} \circ inc_{p_1,in(t)(p_1)} \circ \cdots \circ inc_{p_{n'},in(t)(p_{n'})}(m)}$$

- userPredComp($test_{p,in(t)}$) = ???
- userActComp($dec_{p,in(t)}$) = ???

Case Study: Petri nets



Abstract semantics

$$\frac{m \geq in(t)}{m \rightarrow_t m - in(t) + out(t)}$$

$$\frac{\bigwedge_{p \in P} test_{p,in(t)(p)}(m) \quad P = \{p_1, \ldots, p_{n'}\}}{m \rightarrow dec_{p_1,in(t)(p_1)} \circ \cdots \circ dec_{p_{n'},in(t)(p_{n'})} \circ inc_{p_1,in(t)(p_1)} \circ \cdots \circ inc_{p_{n'},in(t)(p_{n'})}(m)}$$

- userPredComp($\text{test}_{\text{p,in(t)}}$) = ???
- userActComp($\text{dec}_{\text{p,in(t)}}$) = ???
- userActComp($\text{inc}_{\text{p,in(t)}}$) = ???

State Space

State Space

- Sorts: nat, pname, mapping

State Space

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- zero: nat

State Space

- Sorts: nat, pname, mapping
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- s: nat $\longrightarrow$ nat

State Space

- Sorts: nat, pname, mapping
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- s: nat $\longrightarrow$ nat
- P1, P2, P3: pname

State Space

- Sorts: nat, pname, mapping
- zero: nat
- s: nat $\longrightarrow$ nat
- P1, P2, P3: pname
- empty: mapping

State Space

- Sorts: nat, pname, mapping
- zero: nat
- s: nat ⟶ nat
- P1, P2, P3: pname
- empty: mapping
- map: pname, nat, mapping ⟶ mapping

State Space

- Sorts: nat, pname, mapping
- zero: nat
- s: nat ⟶ nat
- P1, P2, P3: pname
- empty: mapping
- map: pname, nat, mapping ⟶ mapping
- map(P1, 1, map(P2, 0, map(P3, 5, empty)))

$test_{p,in(t)}$

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- $applyTo(C, S) = ITE(C, S, One_3(applyTo(C, S))$

$test_{p,in(t)}$

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- $checkLoc_p = \{map(p, \$x, \$m) \rightsquigarrow map(p, \$x, \$m)\}$

$\text{test}_{p,in(t)}$

- $\text{applyTo}(C, S) = \text{ITE}(C, S, \text{One}_3(\text{applyTo}(C, S))$
- $\text{checkLoc}_p = \{\text{map}(p, \$x, \$m) \rightsquigarrow \text{map}(p, \$x, \$m)\}$
- $\text{testCond}_n = \{\text{map}(\$p, s(..(s(\$x))), \$m) \rightsquigarrow$
  $\text{map}(\$p, s(..(s(\$x))), \$m)\}$

$test_{p,in(t)}$

- $applyTo(C, S) = ITE(C, S, One_3(applyTo(C, S))$
- $checkLoc_p = \{map(p, \$x, \$m) \rightsquigarrow map(p, \$x, \$m)\}$
- $testCond_n = \{map(\$p, s(..(s(\$x))), \$m) \rightsquigarrow map(\$p, s(..(s(\$x))), \$m)\}$
- $userPredComp(test_{p,n}) = applyTo(checkLoc_p, testCond_n)$

$dec_{p,in(t)}$ & $inc_{p,in(t)}$

$dec_{p,in(t)}$ & $inc_{p,in(t)}$

- $decStrat_n$ = { map($p, s(..(s($x))), $m) ⇝
  map($p, $x, $m) }

$\mathrm{dec}_{p,in(t)}$ & $\mathrm{inc}_{p,in(t)}$

- $\mathrm{decStrat}_n$ = { map($p, s(..(s($x))), $m) ⤳
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- $\mathrm{incStrat}_n$ = { map($p, $x, $m) ⤳
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$dec_{p,in(t)}$ & $inc_{p,in(t)}$

- $decStrat_n$ = { map($p, s(..(s($x))), $m) ⤳
  map($p, $x, $m) }

- $incStrat_n$ = { map($p, $x, $m) ⤳
  map($p, s(..(s($x))), $m)}

- userActComp($dec_{p,n}$) =
  applyTo($checkLoc_p$, $decStrat_n$)

$\mathrm{dec}_{p,in(t)}$ & $\mathrm{inc}_{p,in(t)}$

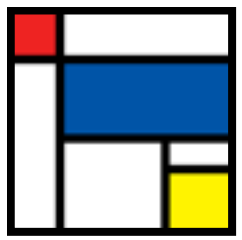
- $\mathrm{decStrat}_n$ = { map($p, s(..(s($x))), $m) ⤳
  map($p, $x, $m) }

- $\mathrm{incStrat}_n$ = { map($p, $x, $m) ⤳
  map($p, s(..(s($x))), $m)}

- userActComp($\mathrm{dec}_{p,n}$) =
  applyTo($\mathrm{checkLoc}_p$, $\mathrm{decStrat}_n$)

- userActComp($\mathrm{inc}_{p,n}$) =
  applyTo($\mathrm{checkLoc}_p$, $\mathrm{incStrat}_n$)

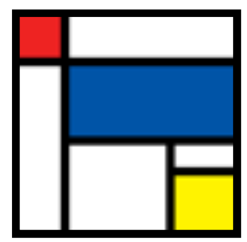
Case Study: Divine



Abstract semantics

$$\frac{c_1 \wedge c_2 \wedge \ldots \wedge c_n}{m \rightarrow a_1 \circ \ldots \circ a_m(m)}$$

Case Study: Divine



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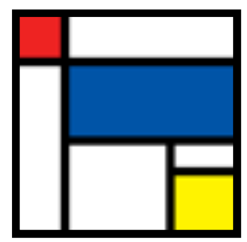

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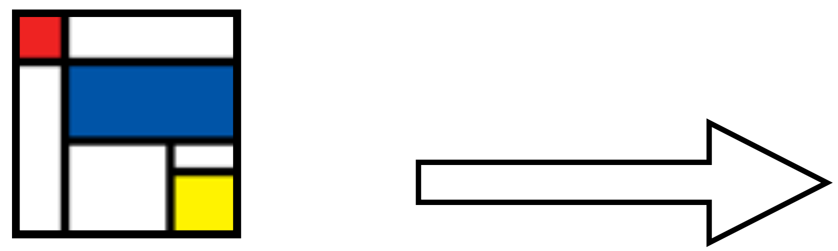

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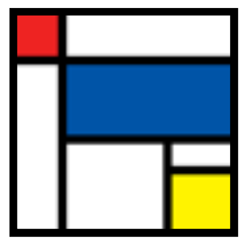

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State Space

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- **trans** v’ = 1 + v; v := v + 1

$$\frac{test \circ eq \circ read_{v'} \circ add \circ read_v \circ push_1(\langle st, m \rangle)}{\langle st, m \rangle \rightarrow write_v \circ add \circ read_v \circ push_1(\langle st, m \rangle)}$$

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Translation to Simple SOS Rules

$$
\begin{aligned}
lComp &: DivBasicExpr \rightarrow Cond \cup Act \\
&: c \ \mathbf{and} \ c' \mapsto lComp(c) \wedge lComp(c) \\
&: true \mapsto true \\
&: false \mapsto false \\
&: e = e' \mapsto test \circ eq \circ lComp(e) \circ lComp(e') \\
&: e + e' \mapsto add \circ lComp(e) \circ lComp(e') \\
&: e - e' \mapsto subt \circ lComp(e) \circ lComp(e') \\
&: id := e \mapsto write_v \circ lComp(e) \\
&: a; a \mapsto lComp(a) \circ lComp(a) \\
&: n \mapsto push_n \\
&: v \mapsto read_v
\end{aligned}
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- userPredComp(test) = test(true, \$m) ⇝ \$m

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- userPredComp(test) = test(true, $m) ⇝ $m

- equals = { eq(0, 0) ⇝ true,
  eq(s($n1), s($n2)) ⇝ eq ($n1, $n2),
  eq(0, s($n1)) ⇝ false,
  eq(s($n1), 0) ⇝ false }

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- RewriteSet(S) = Choice(Union(S, Not(S)),
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- RewriteSet(S) = Choice(Union(S, Not(S)),
  Choice(S, Not(S)))

- applyEquals = One_1(Fixpoint(RewriteSet(equals)))

readV

```
checkV = map(j, $n, $s) ⇝ map(j, $n, $s)
findVAndApply(S) = ITE(checkV, S,
                       One3(findAndApply(S)))
upSwap = map($v, $n, map(stackH, $n, $s)) ⇝
          map(stackH, $n, map($v, $n, $s))
upAux = Choice(upSwap,
         Sequence(One3(upAux), upSwap))
endUp = map(stackH, $n, map($v, $n, $s)) ⇝
          map(t, $n, map($v, $n, $s))
up = Choice(endUp, upAux)
copy = map(j, $n, $p) ⇝ map(stackElt,
         $n, map(j, $n, $p))
readV = Sequence(findVAndApply(copy), up)
```

Practical results

Kanban problem

Practical results

Kanban problem

- Small Petri net

Practical results

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- 16 places & 16 transitions, marking changes with scale parameter

Practical results

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- State space for scale parameter 100

Practical results

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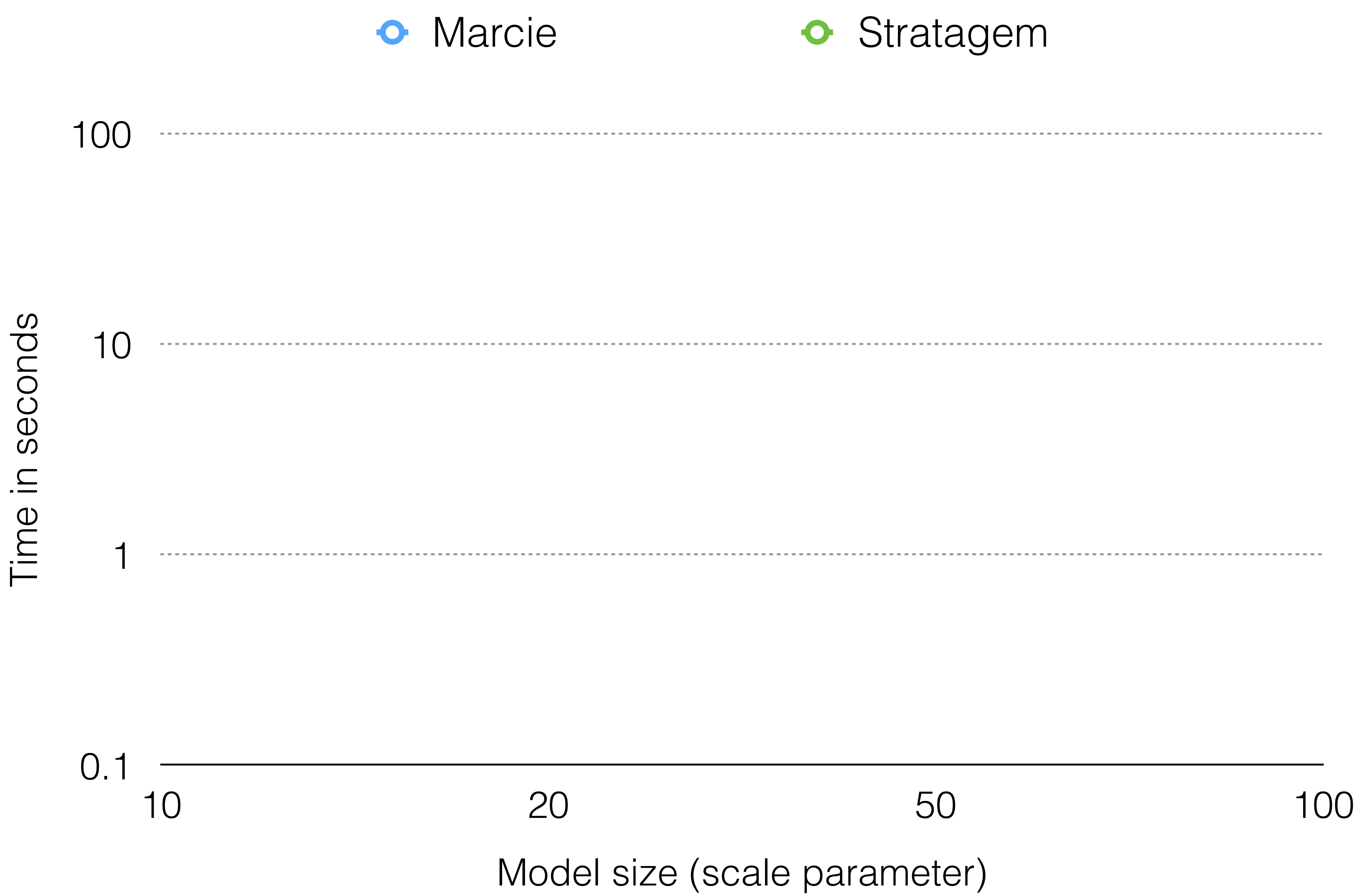
  - $1.7263 \cdot 10^{19}$ states

Practical results

Kanban problem



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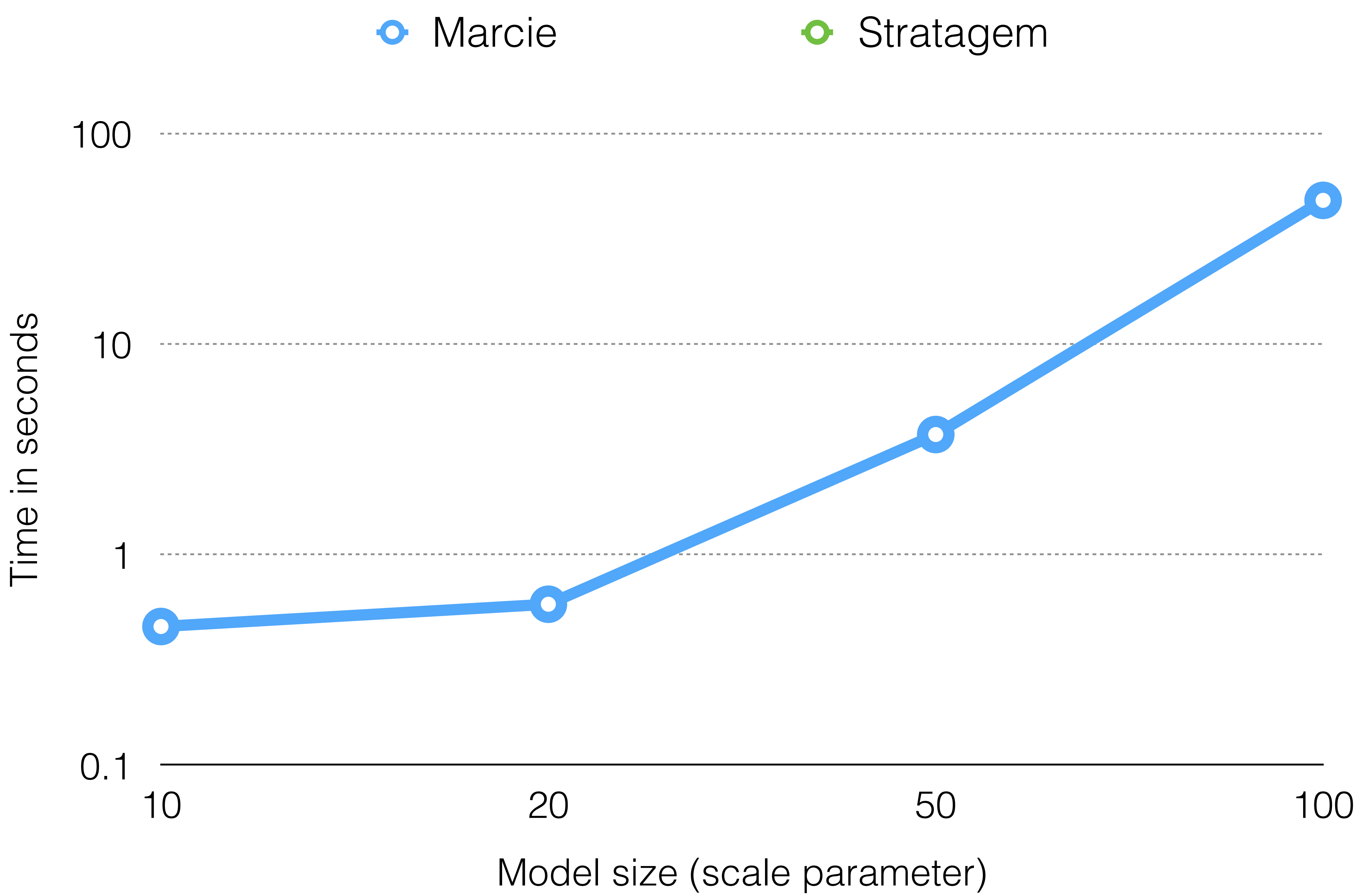

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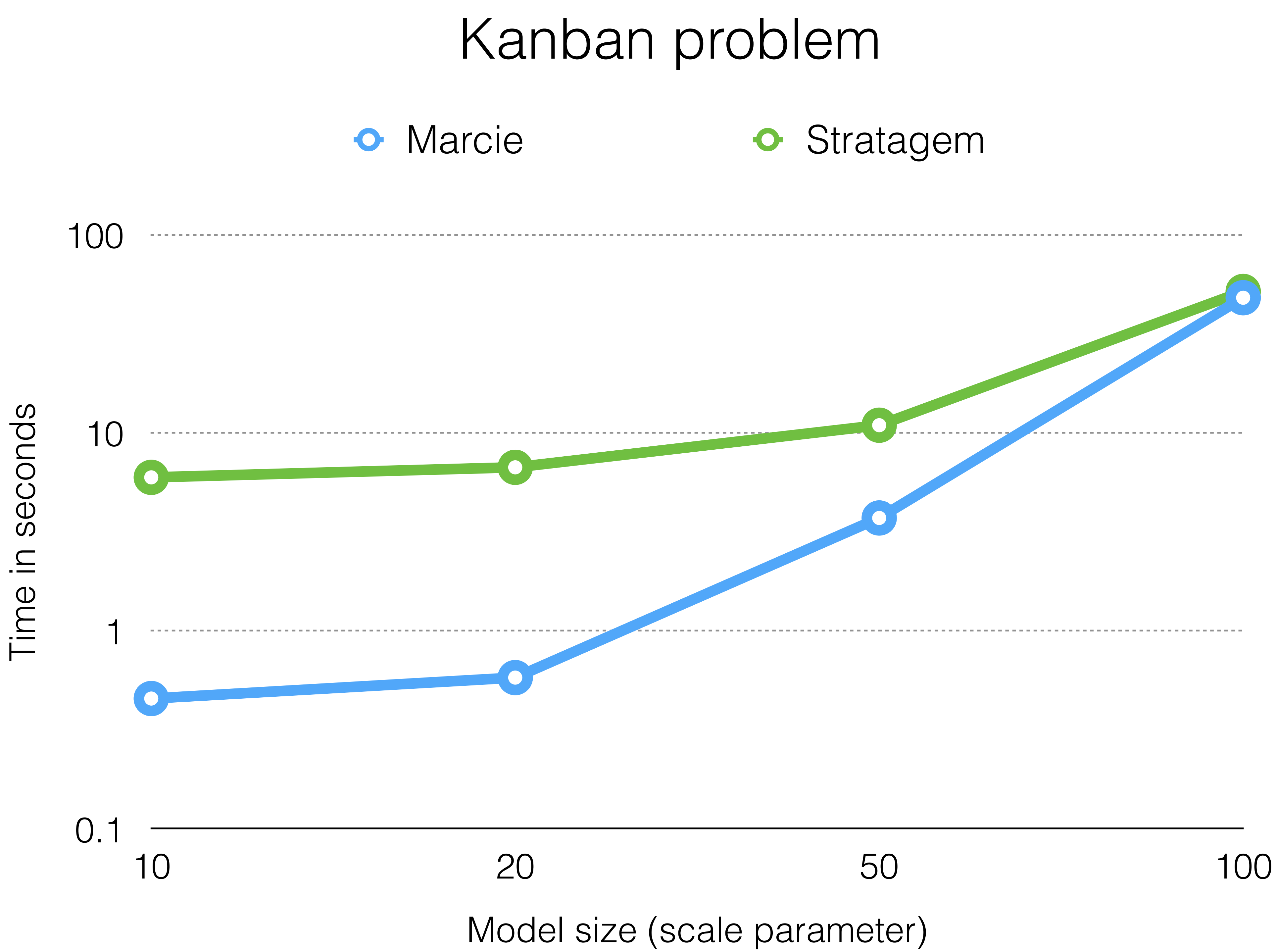

Practical results

Sharedmem problem

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- State space for scale parameter 50

Practical results

Sharedmem problem

- Petri net's places and transition increase with scale parameter
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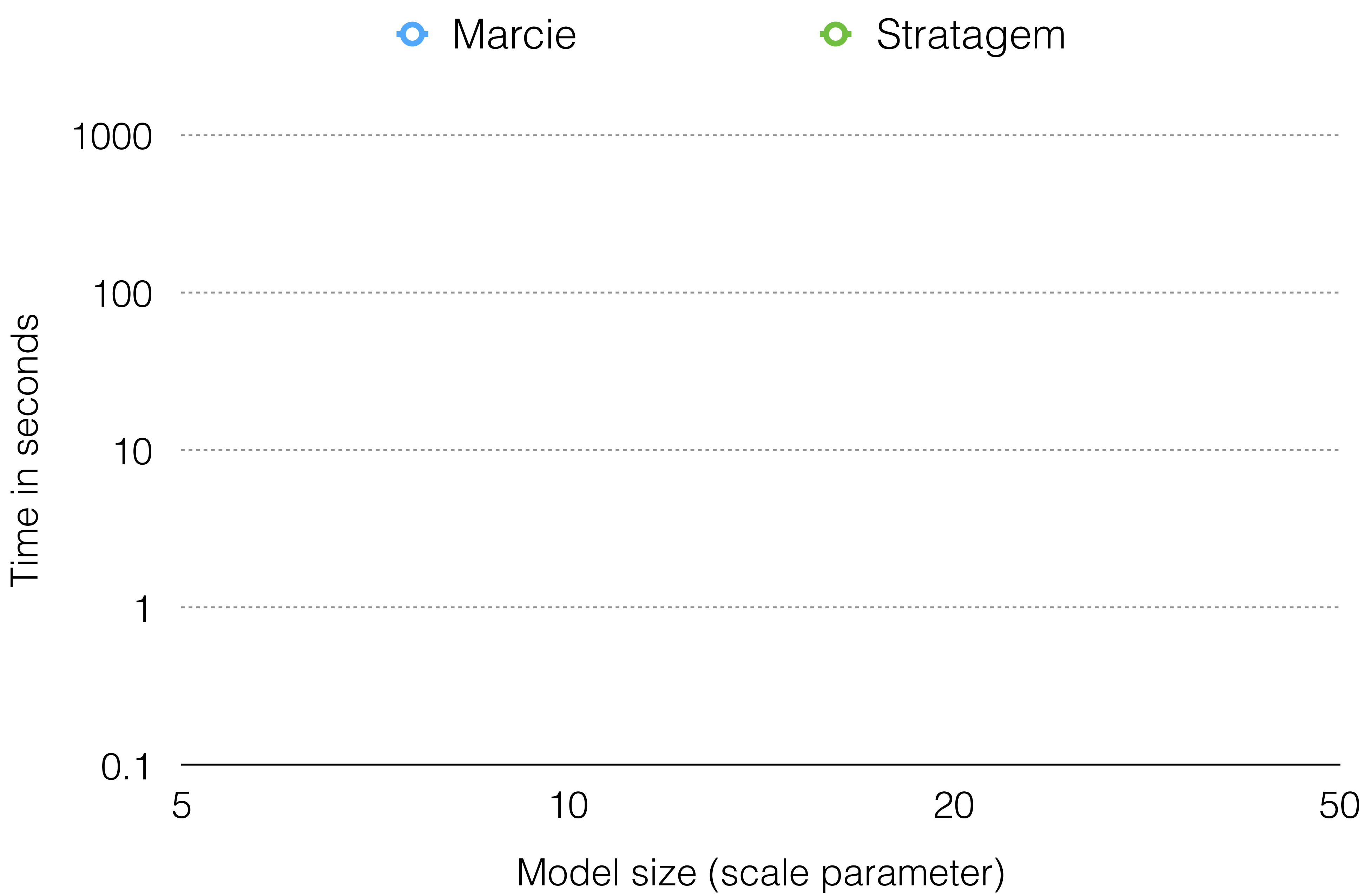
- State space for scale parameter 50
    - $5.87 \cdot 10^{26}$ states

Practical results

SharedMem problem



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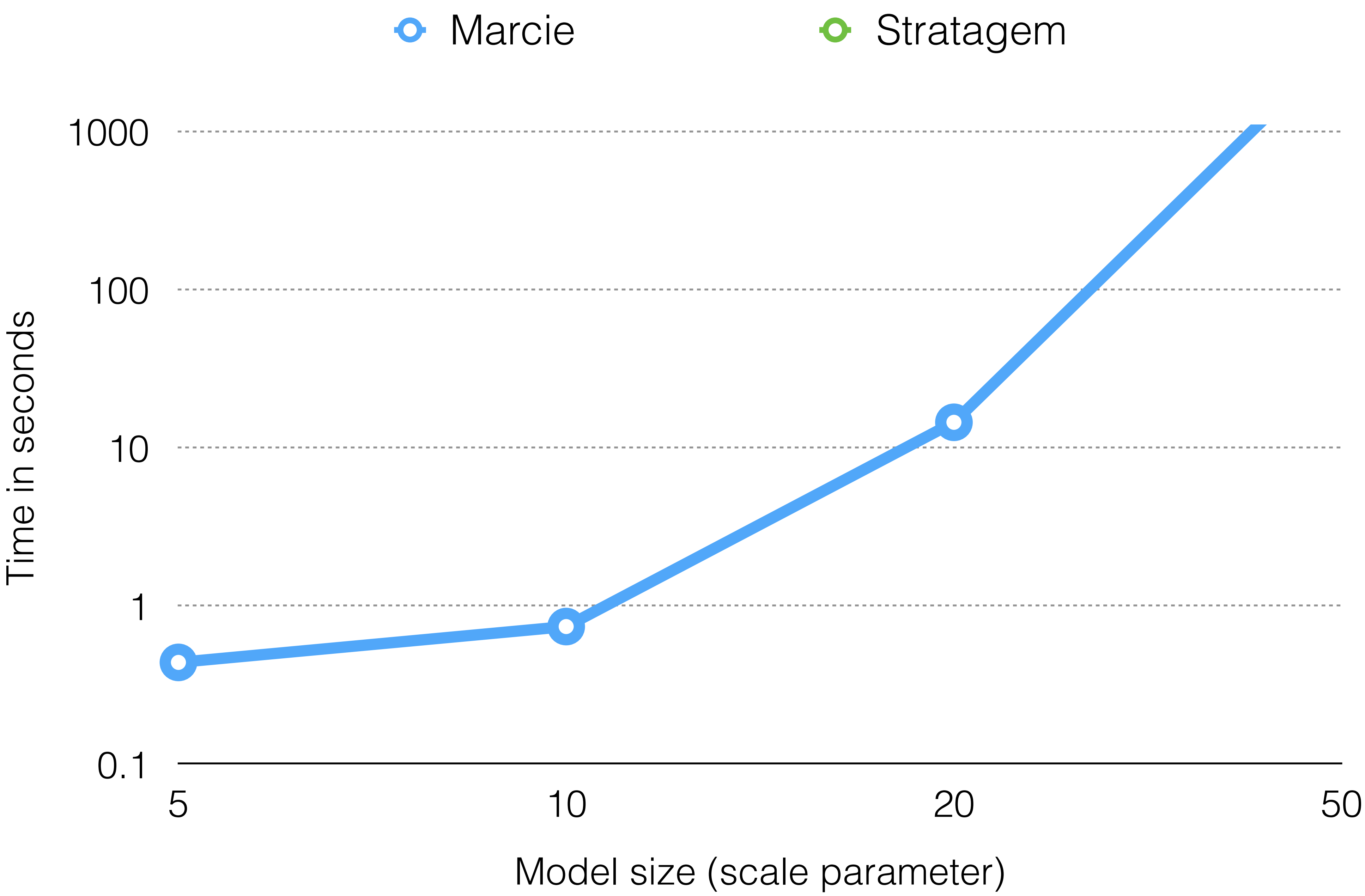

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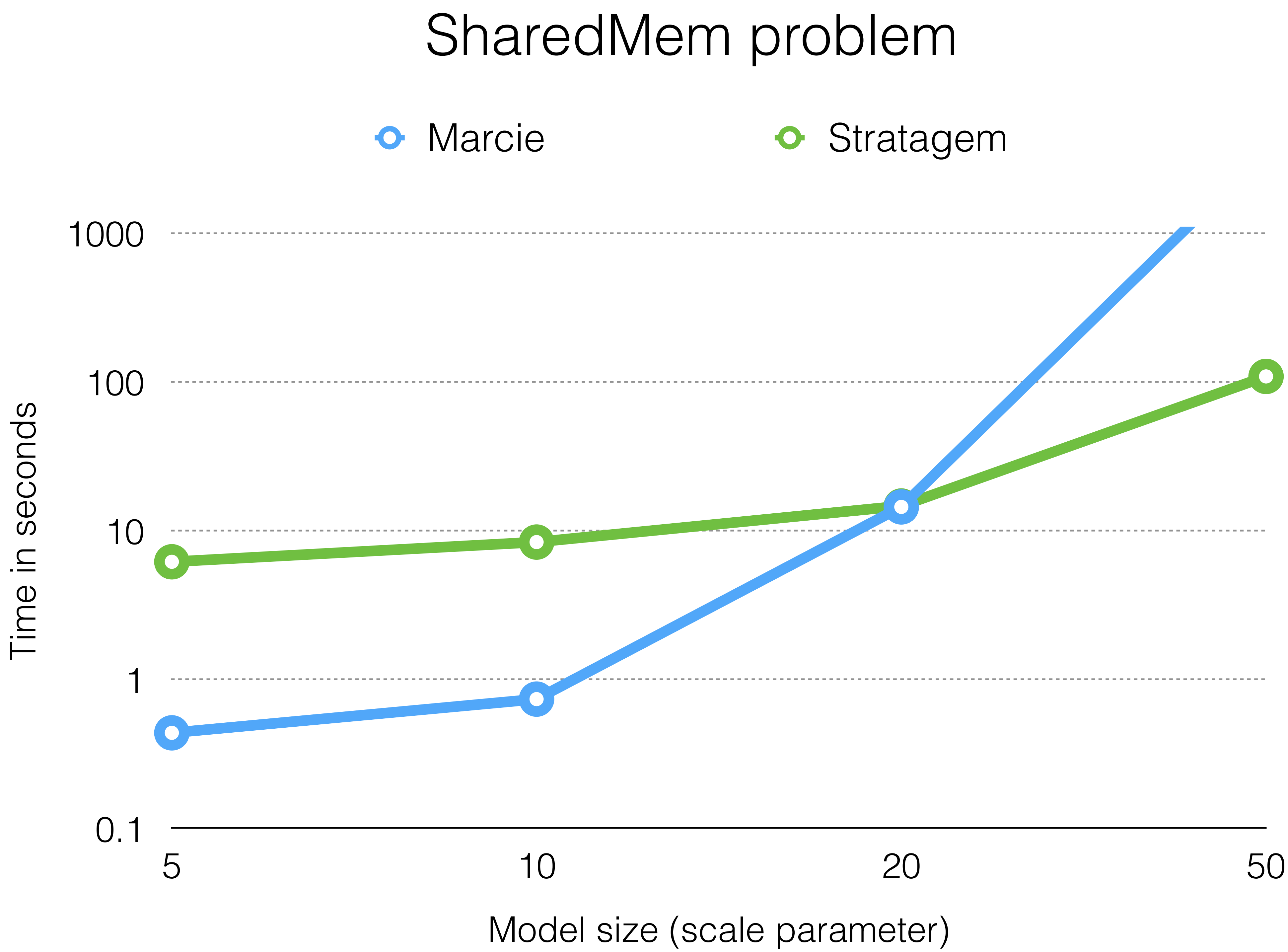

Set rewriting

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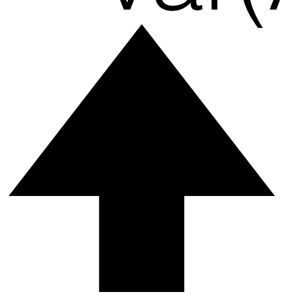
var(A, 1, var(B, 2, var(C, 0, empty)))



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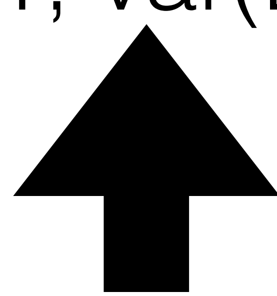

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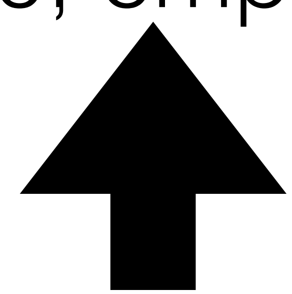

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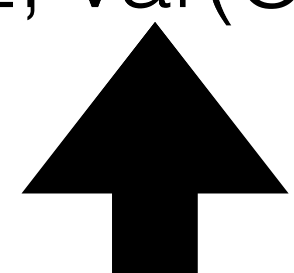

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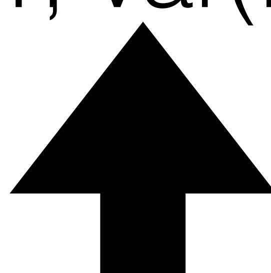

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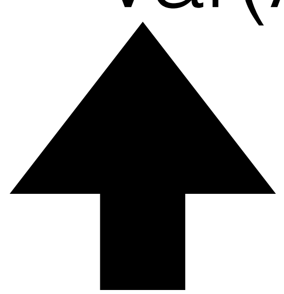

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Conclusion

- General translation
- Works well with common language constructions
- Efficient implementation