Cost Linear Temporal Logic for Verification

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Quantitative Verification

Qualitative Verification

Yes-No Question

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Question has an answer in a domain D

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- D continuous (time, probability) $\Rightarrow$ dedicated models
  timed, hybrid, stochastic ...

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  - yes, but inefficient

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- D continuous (time, probability) $\Rightarrow$ dedicated models timed, hybrid, stochastic ...
- D discrete $\Rightarrow$ same models as qualitative setting
  - yes, but inefficient

We focus on *discrete quantitative*

Motivating Example

Dining philosophers (ressource sharing model)

- N philosophers, N forks
- philo either thinking, or eating
- 2 forks needed to eat
- Starvation = does a philo not eat forever?

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- 2 forks needed to eat
- Starvation = does a philo not eat forever?

In real life, starvation occurs in finite time

- How long a philo thinks? (discrete time)
- Bounds on $\Rightarrow$ max/min thinking time (per philo, globally ...)

Motivating Example

Instrumentation: How to measure thinking (logical) time?

add a variable timer_p

```
if (philo p thinks) ++timer_p;
if (philo p eats) timer_p = 0;
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Qualitative Verification of the instrumented model.

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Qualitative Verification of the instrumented model.

Numerous Drawbacks

- model modification = strong semantical risk
- translate quantitative to qualitative: values known a priori

Objective

Verification principle: model and property are independent
Quantitative measure should remain in the logic.

Proposal

Use a quantitative logic towards “discrete quantitative” verification

- Linear Temporal Logic very popular
- extend LTL with counting

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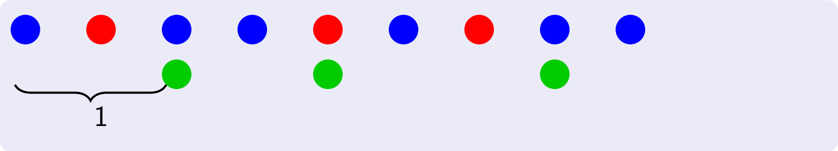

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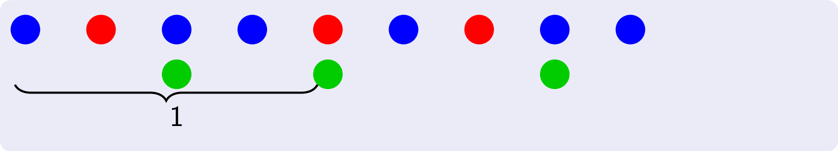

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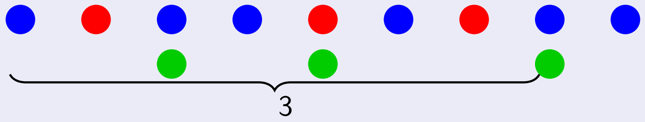

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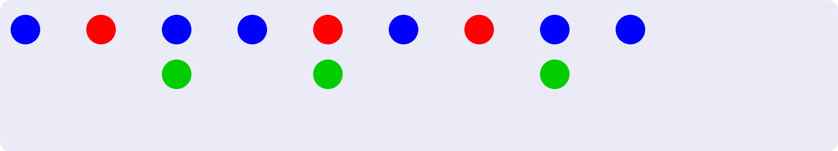

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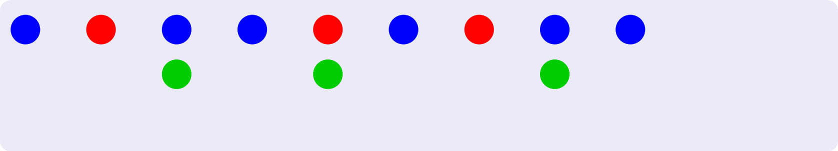

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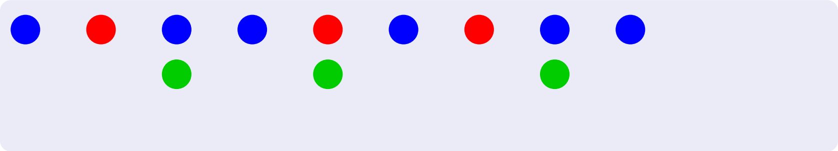

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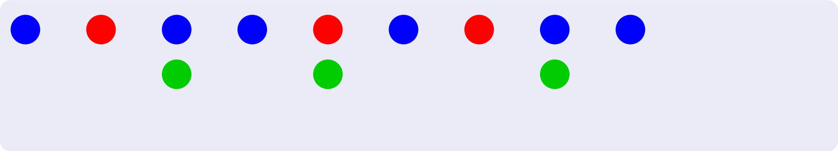

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- $[\![\phi]\!]_{\leq} = \inf\{\text{possible } n\}$

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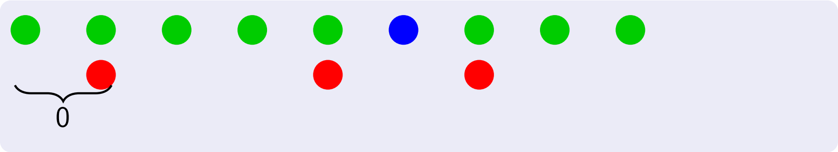

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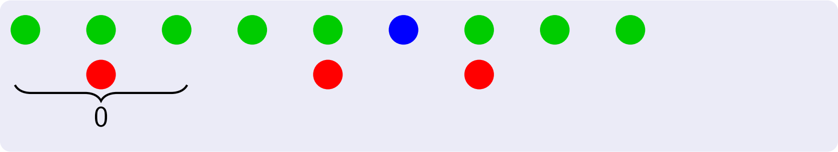

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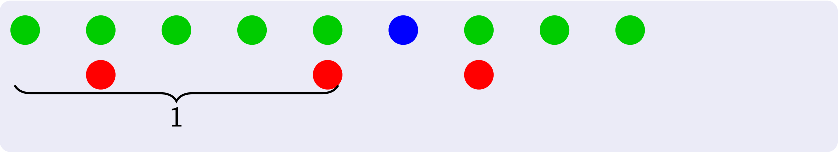

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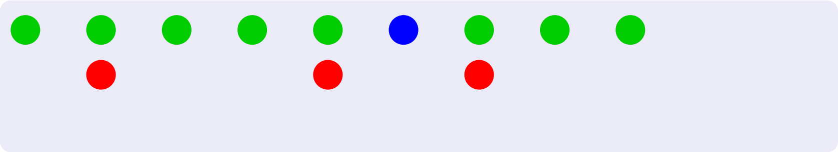

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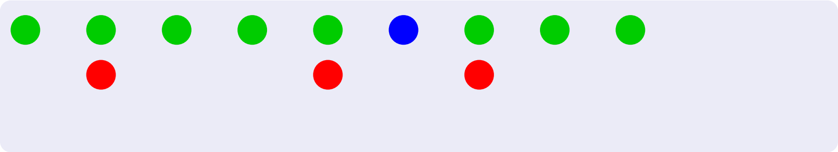

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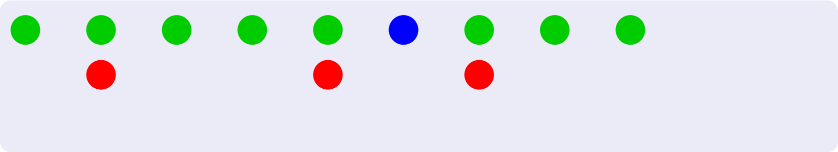

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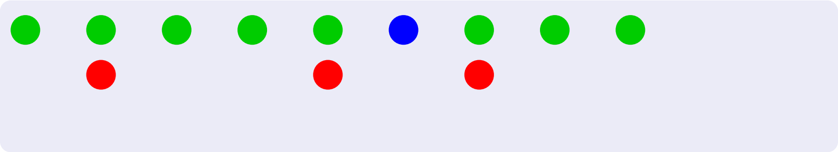
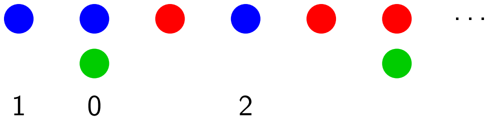


- find a n dominated by $\#$ of
- $\{$possible $n\}$ $\downarrow$-closed
- same n for all occurrences of **R**$^{>}$ in the formula
- $[\![\phi]\!]_{>} = \sup\{$possible $n\}$

Example

$\mathbf{G}\,(\textcolor{blue}{\bullet} \implies (\perp\ \mathbf{U}^{\leq}\ \textcolor{green}{\bullet}))$

counts the minimal distance between any $\textcolor{blue}{\bullet}$ and the next $\textcolor{green}{\bullet}$



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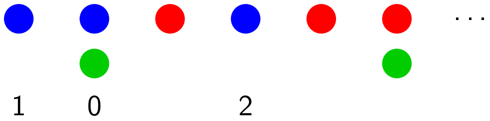


$(u, n) \models \phi$ iff n dominates every $\mathbf{U}^{\leq}$

- here $(u, n) \models \phi$ iff $n \geq 2$

$[\![\phi]\!]_{\leq}(u) = \inf\{\text{possible } n\} = 2$

Negation and Duality

in $LTL^{\leq}$ and $LTL^{>}$, cost operators cannot be negated

Remark

$$(u,n) \models a \, \mathbf{U}^{\leq} \, b \quad \iff \quad (u,n) \not\models \neg a \, \mathbf{R}^{>} \, \neg b$$

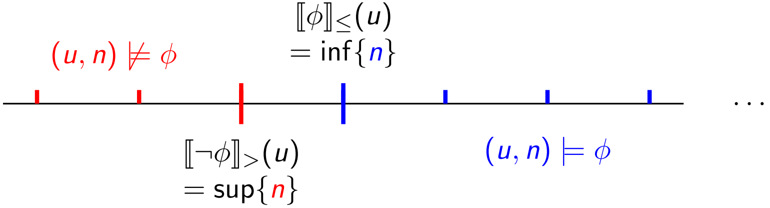
Duality through negation

$$LTL^{\leq} \underset{\neg}{\overset{\neg}{\rightleftharpoons}} LTL^{>}$$

and push negations to leaves (Negative Normal Form)

Negation and Semantics

$$(u, n) \models \phi \iff (u, n) \not\models \neg\phi$$



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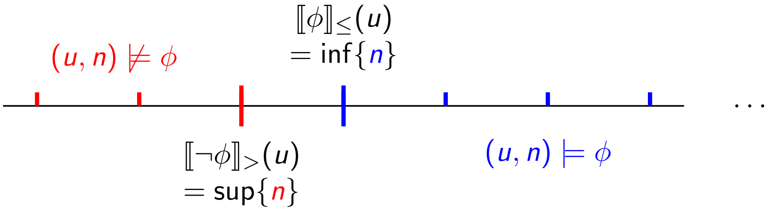
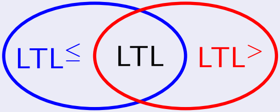


- semantically, $\neg$ is ± 1

And LTL?

Syntactically



Semantically ($\phi \in LTL$)

$$u \vdash \phi \iff \forall n \in \mathbb{N}.(u, n) \models \phi$$

$LTL^{\leq}$	LTL	$LTL^{>}$
$[\![\phi]\!]_{\leq}(u) = 0$	$u \vdash \phi$	$[\![\phi]\!]_{>}(u) = +\infty$
$[\![\phi]\!]_{\leq}(u) = +\infty$	$u \not\vdash \phi$	$[\![\phi]\!]_{>}(u) = 0$

$\mathrm{LTL}^{\leq}$ and $\mathrm{LTL}^{>}$ model-checking

$\mathrm{LTL}^{\leq}$ and $\mathrm{LTL}^{>}$ more expressive than LTL
New problems arise

- $(u, n) \models \phi$? for a given $n \in \mathbb{N}$
  - $\exists u$
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- is $[\![\phi]\!]_{\leq}$ bounded over a given language L?
  - decidable when L regular [Bojańczyk and Colcombet, 2006]

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  - $\exists u$
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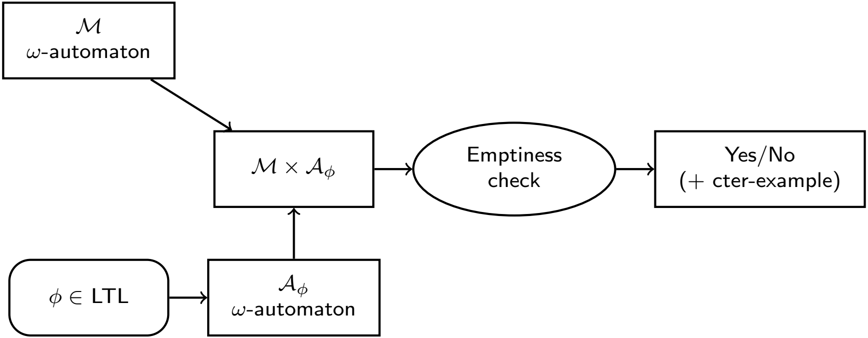
- is $[\![\phi]\!]_{\leq}$ bounded over a given language L?
  - decidable when L regular [Bojańczyk and Colcombet, 2006]
- actual values of bounds over a given set L
  - $\sup_L [\![\phi]\!]_{\leq}$ ($\mathrm{LTL}^{>}$ seems appropriate)
  - $\inf_L [\![\phi]\!]_{\leq}$ ($\mathrm{LTL}^{\leq}$ seems appropriate)

Following the Automata Approach: LTL



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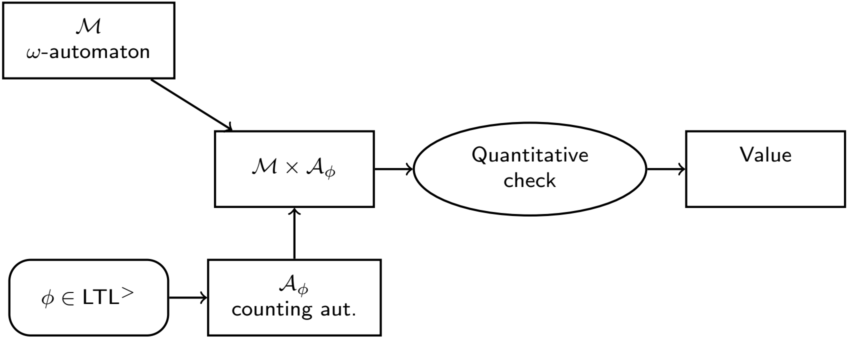
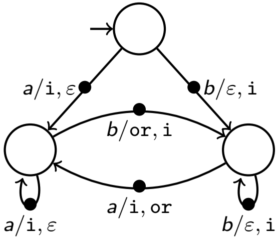


Cost Register Automata

[Bojańczyk and Colcombet, 2006]

Transition-Based Generalized Büchi Automaton
+ finite set of counters
actions on counter: observe (o), increment (i), reset (r), ε



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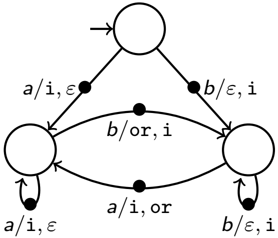

2 counters

- 1st one counts consecutive a's
- 2nd one counts consecutive b's

$$Val(\rho) = \min\{\text{observed values}\}$$
$$= \min \# \text{ consecutive occurrences of the same letter}$$

Cost Register Automata Semantics

$>$-automata

- $\mathrm{Val}_>(\rho) = \inf\{\text{observed counter values}\}$
- $[\![\mathcal{A}]\!]_>(u) = \sup_{\rho \text{ acc. run on } u} \mathrm{Val}_>(\rho)$

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$\leq$-automata

- $\mathrm{Val}_{\leq}(\rho) = \mathsf{sup}\{\text{observed counter values}\}$
- $[\![\mathcal{A}]\!]_{\leq}(u) = \mathsf{inf}_{\rho \text{ acc. run on } u} \mathrm{Val}_{\leq}(\rho)$

actions: `io`, `r`, ε

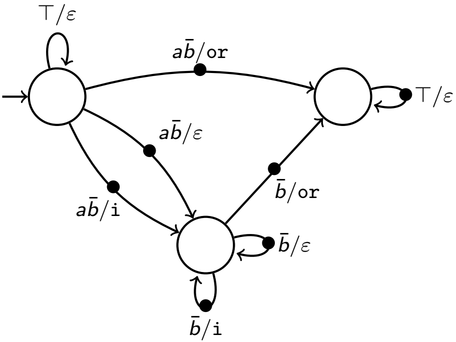
From LTL$^{>}$ to $>$-automata

Translation idea [Kuperberg, 2012]

- similar to LTL $\rightarrow$ Büchi translation
- accepting conditions on the transitions
- one counter for each $\mathbf{R}^{>}$
  - counts the occurrences of lhs of $\mathbf{R}^{>}$, as expected

Automaton for $\phi = \mathbf{G}(a \implies \bot \mathbf{U}^{\leq} b)$

$[\![\phi]\!]_{\leq}(u) =$ max distance between any a and the next b



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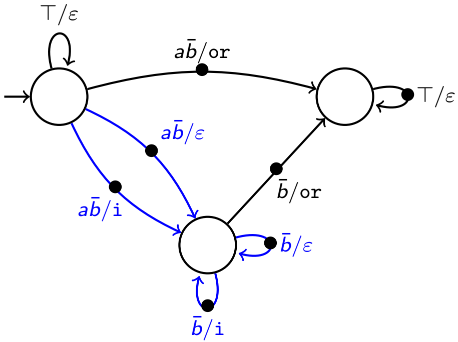

- non-determinism

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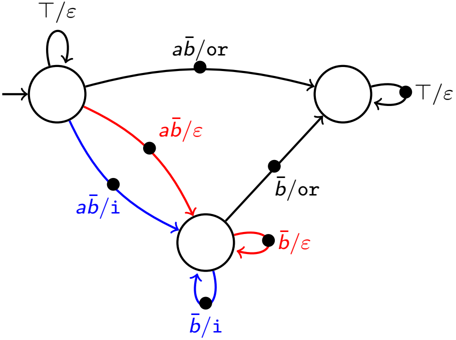

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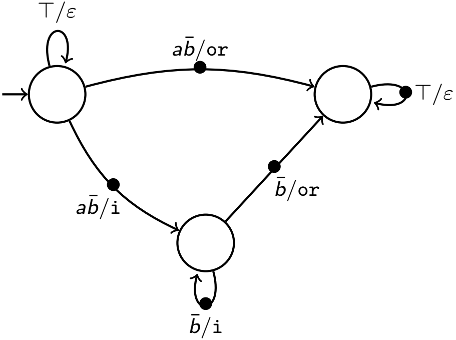

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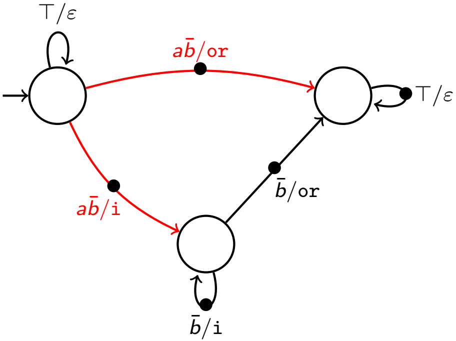

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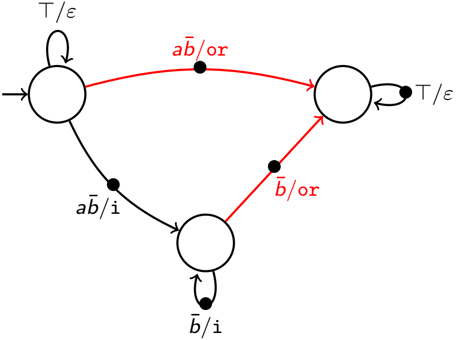

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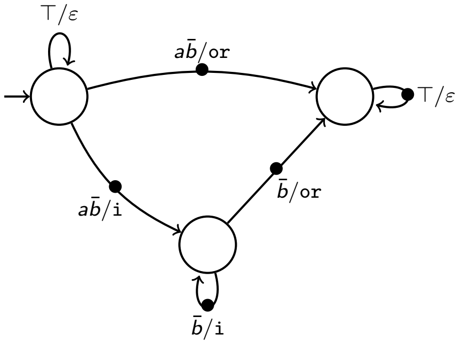

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- guess the a for which the max is reached
- guess: `or` just before b
- wrong guesses yield runs with smaller values, eliminated by sup in $[\![\mathcal{A}]\!]_{>}$

Compute the Upper Bound of a $>$-Automata

Several sub-problems.

Boundedness

Is $\sup[\![\mathcal{A}]\!]_>$ finite?

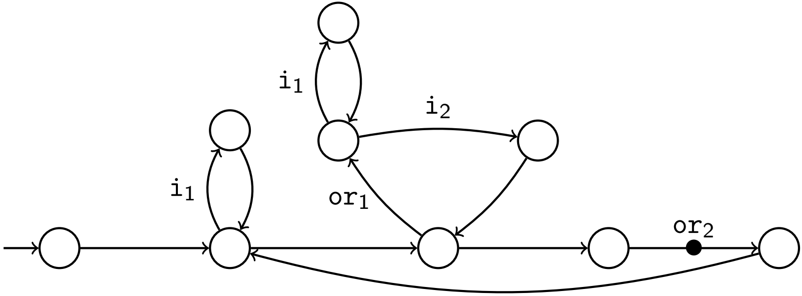
Exact value

If finite, value of $\sup[\![\mathcal{A}]\!]_>$?

Boundedness [Colcombet, 2009]

Property

sup$[\![\mathcal{A}]\!]_{>} = \infty$ iff $\exists \rho$ acc. run s.t.
every $\texttt{or}_\gamma$ is preceded by a cycle that increments γ and never $\texttt{or}$ it (γ-cycle).



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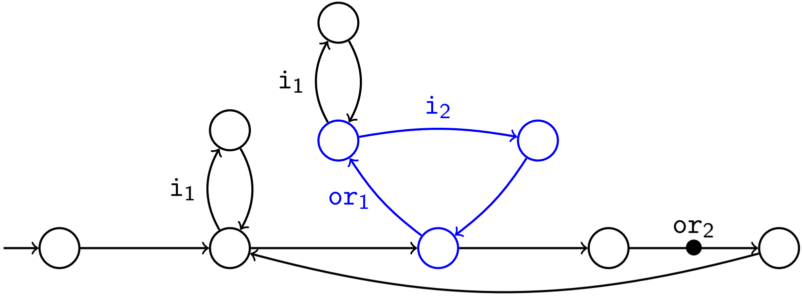

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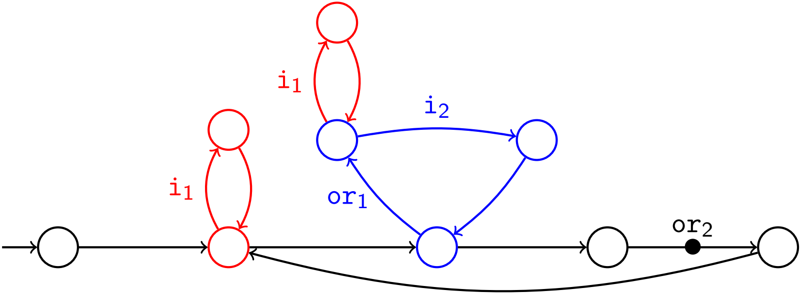

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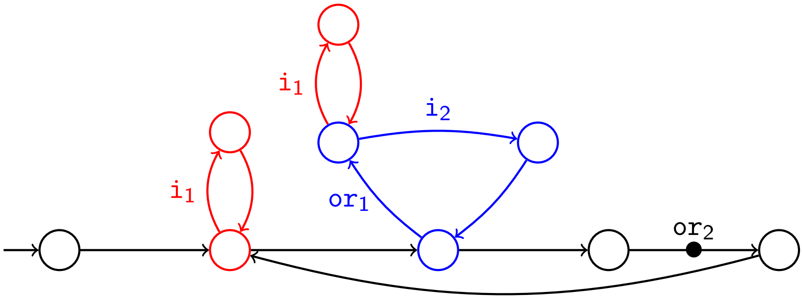

Upper bound

if bounded, sup$[\![\mathcal{A}]\!]_> \leq |Q_{\mathcal{A}}|$

Back to LTL

Büchi Emptiness Check

$u \in L(\mathcal{A})$ iff $\exists$ acc. run on u
possible early answer

Bound Computation

$\text{sup}[\![A]\!]_{>} = \text{sup}_{\text{all acc. runs } \rho} \mathrm{Val}_{>}(\rho)$
no early break is possible
but once a candidate n is found, all values $< n$ are ruled out

Back to LTL

Büchi Emptiness Check

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Problem: remove runs of value $< n$ in $\mathcal{A}$

Back to LTL

Problem

INPUT: $\phi \in \mathrm{LTL}^{>}, n \in \mathbb{N}$

OUTPUT: $\phi[n] \in \mathrm{LTL}$ s.t. $u \vdash \phi[n] \iff [\![\phi]\!]_{>}(u) \geq n$

n is "hardcoded" in $\phi[n]$

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- for $\phi, \psi \in \mathrm{LTL}$
  - $(\phi\ \mathbf{R}^{>}\ \psi)[0] \equiv \phi\ \mathbf{R}\ \psi$
  - $(\phi\ \mathbf{R}^{>}\ \psi)[n] \equiv (\phi \wedge \mathbf{X}\ (\phi\ \mathbf{R}^{>}\ \psi)[n-1])\ \mathbf{R}\ \psi$
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$\phi = a\ \mathbf{R}^{>}\ b,\ n = 1$

$(a \wedge \mathbf{X}\ (a\ \mathbf{R}\ b))\ \mathbf{R}\ b$

n is "hardcoded" in $\phi[n]$

LTL to remove runs

Recall

$[\![\phi_1 \wedge \phi_2]\!]_> = \sup\{n \text{ possible for } \phi_1 \text{ AND } \phi_2\} = \min([\![\phi_1]\!]_>, [\![\phi_2]\!]_>)$

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If $n \leq \sup[\![\phi]\!]_>$ then $\sup[\![\phi \wedge \phi[n]]\!]_> = \sup[\![\phi]\!]_>$

Refinement loop for upper bound

Repeat until $\mathcal{A}_\phi$ has no more accepting runs

Find an acc. run ρ in $\mathcal{A}_\phi$
Let $n = \mathrm{Val}_>(\rho)$
Then $n \leq \sup[\![\mathcal{A}]\!]_>$
Update $\phi \leftarrow \phi \wedge \phi[n]$ and restart

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Requires $\sup[\![A_{\phi_0}]\!]_> < \infty$

$\sup[\![A_{\phi_0}]\!]_> = \sup_{\rho \text{ acc. run}} \mathrm{Val}_>(\rho)$
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- easy to implement: formulae manipulations + Büchi EC
- refinement: less and less behaviors

Loop for boundedness + exact value

```
INPUT: φ_0 and M
```

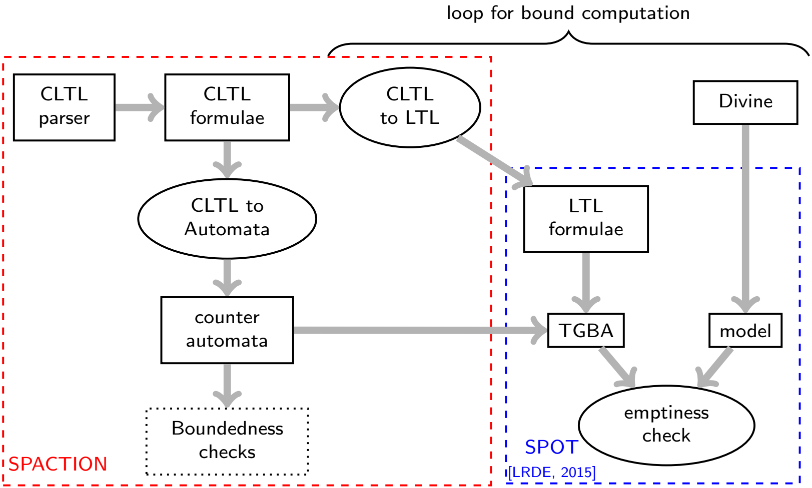
INPUT: ϕ_0 **and** $\mathcal{M}$

$B \leftarrow |\mathcal{A}_{\phi_0}| \times |\mathcal{M}|$
// *if finite*, $\sup[\![\mathcal{A}_{\phi_0}]\!]_> \leq |\mathcal{A}_{\phi_0} \times \mathcal{M}| \leq B$
$\phi \leftarrow \phi_0$
$n \leftarrow 0$

while $(\mathcal{L}(\mathcal{A}_\phi \times \mathcal{M}) \neq \emptyset)$ {
  $\rho \leftarrow$ an accepting run in $\mathcal{A}_\phi \times \mathcal{M}$
  $n \leftarrow \mathrm{Val}_>(\rho)$

  if $(n > B)$
    return unbounded
  else
    $\phi \leftarrow \phi_0 \wedge \phi_0[n]$
}
return n

Our tool Spaction



Conclusion

CLTL = nice extension of LTL

- expressivity
- counter automata = nice extension of ω-automata
- separate the logics from the model
- algorithm for bound computation
  - relies on ω-automata emptiness check
- working tool (in progress)

What's next?

- examples and use cases
- more abstractions and refinements
  - boundedness: smaller automata, fewer counters
  - exact value: smaller synchronized products
- relaxed versions: $\sup[\![\mathcal{A}_1]\!]_> \leq n * \sup[\![\mathcal{A}_2]\!]_>$
- variants and generalization

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