

Combining Explicit and Symbolic Approaches for Better On-the-Fly LTL Model Checking

Kais Klai (LIPN/Paris 13)

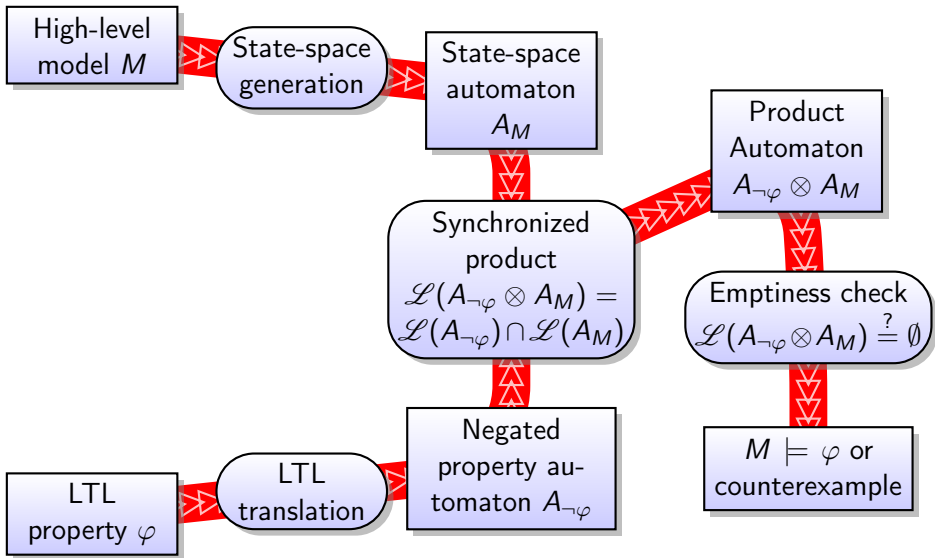
Joint work with A. Duret-Lutz, D. Poitrenaud and Y. Thierry-Mieg

MeFoSyLoMa 4 Novembre 2011

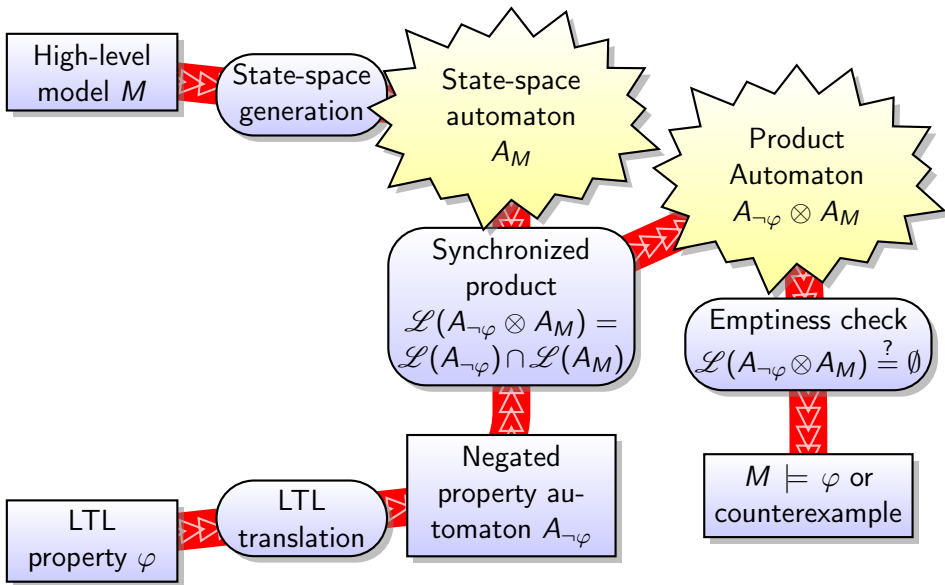
Outline

- 1 Automata-Theoretic Explicit LTL Model Checking
- 2 Explicit Approach
- 3 Fully Symbolic
- 4 Hybrid approaches
 - SOG
 - SOP
 - SLAP
 - SLAP-FST
- 5 Summary
- 6 Experiments
- 7 Results
- 8 Conclusion and Perspectives

Automata-Theoretic Explicit LTL Model Checking



Automata-Theoretic Explicit LTL Model Checking



Automata-Theoretic Explicit LTL Model Checking

High-level
model M

On-the-fly generation
of state-space automaton
 A_M

Product
Automaton
 $A_{\neg\varphi} \otimes A_M$

Synchronized
product
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) =$
 $\mathcal{L}(A_{\neg\varphi}) \cap \mathcal{L}(A_M)$

Emptiness check
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) \stackrel{?}{=} \emptyset$

LTL
property φ

LTL
translation

Negated
property au-
tomaton $A_{\neg\varphi}$

$M \models \varphi$ or
counterexample

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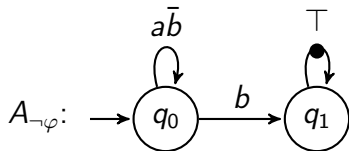
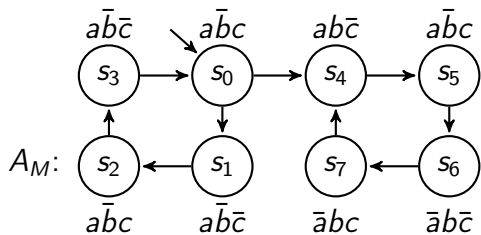
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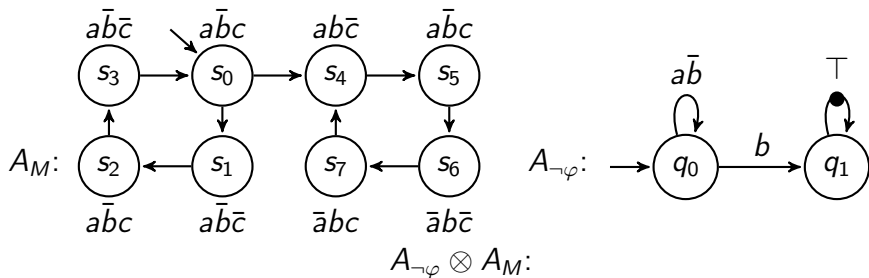
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property au-
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Explicit Approach



Explicit Approach



- Emptiness check = (on-the-fly) emptiness check of the product
- State explosion problem

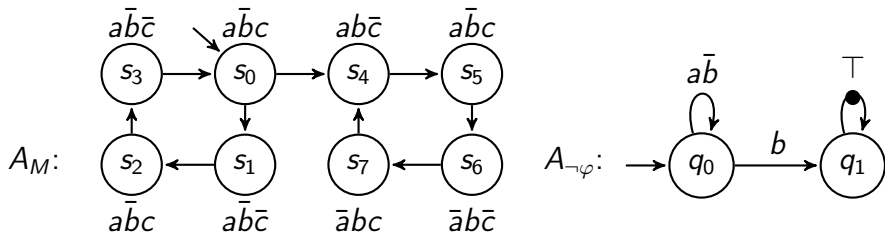
Fully Symbolic Approach (FS)

- Encode the Kripke structure and the property automaton using BDDs.
- Combine the two BDDs. (Symbolic product.)
- Use fixpoint computations to decide whether the product contains an accepting cycle.
- Two symbolic emptiness checks compared:
 - EL
 - OWCTY
- Emptiness check is not on-the-fly
- No suttering invariant reductions

Hybrid approaches

- Goal : combine explicit and symbolic approaches
- Get the best of both worlds
- Explicit graphs with symbolic nodes (aggregates)
- The system
 - 1 SOG: event-based logic (Phd Thesis Kais Klai 2003, S. Haddad, J-M. Ilié and K. Klai ATVA'04)
 - 2 SOG: state-based logic (K. Klai and D. Poitrenaud, PN'08)
- The product:
 - Symbolic Observation Product (SOP)
 - Self Loop Aggregation Product (SLAP)
 - SLAP - Fully Symbolic search in Terminal states (SLAP-FST)

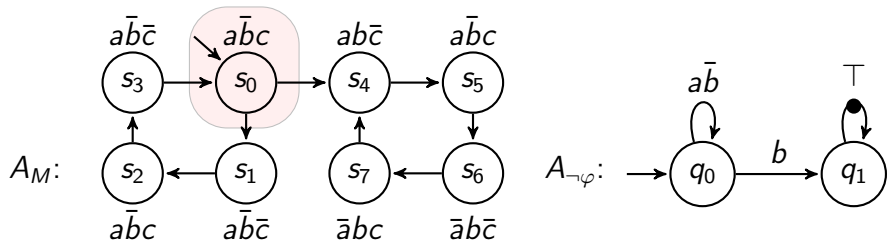
Symbolic Observation Graph (SOG)



Symbolic Observation Graph:

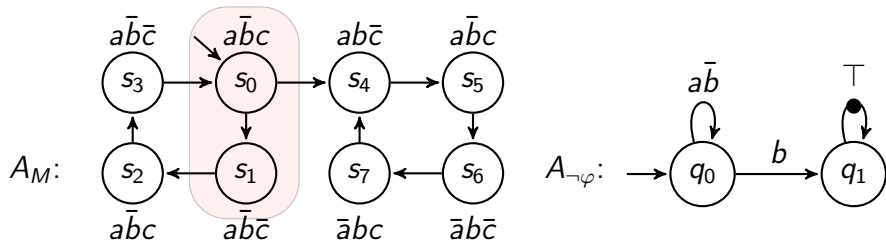
- For stuttering invariant properties (e.g., $LTL \setminus X$)
- Ignore non-observable propositions
- Aggregate Kripke states with homogeneous labels
- Represent aggregates using BDDs

Symbolic Observation Graph (SOG)



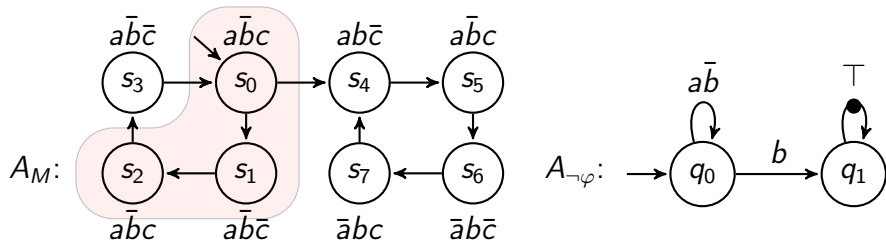
Symbolic Observation **Graph**:

Symbolic Observation Graph (SOG)



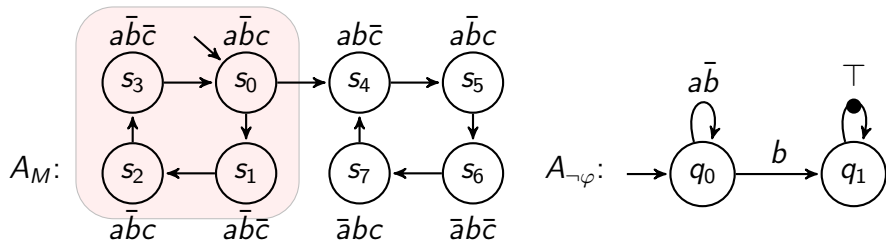
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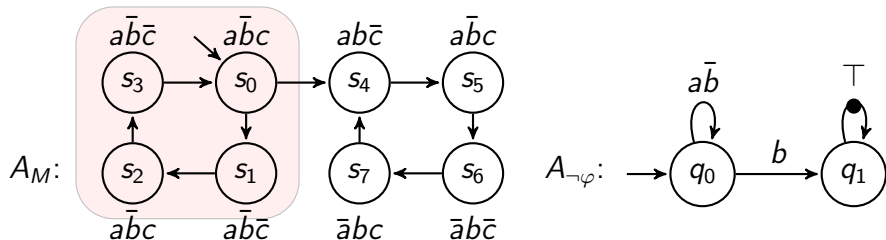
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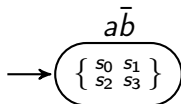


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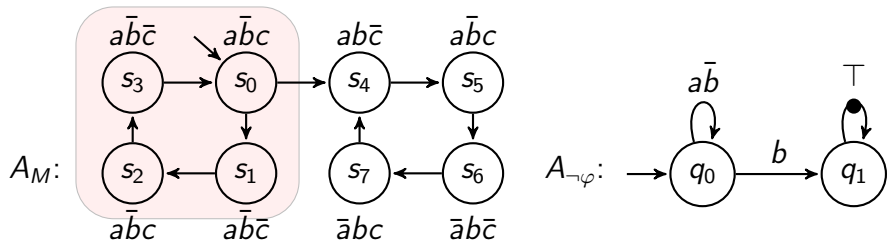
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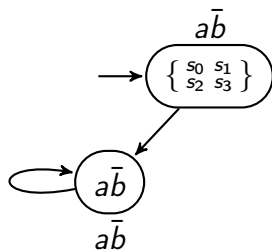
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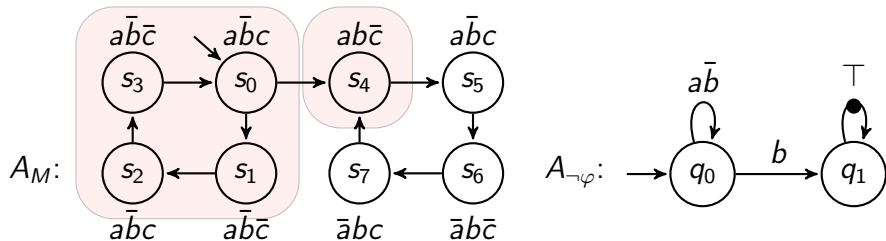
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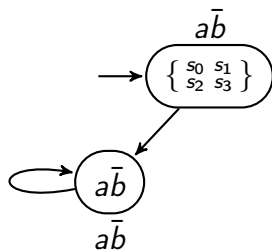
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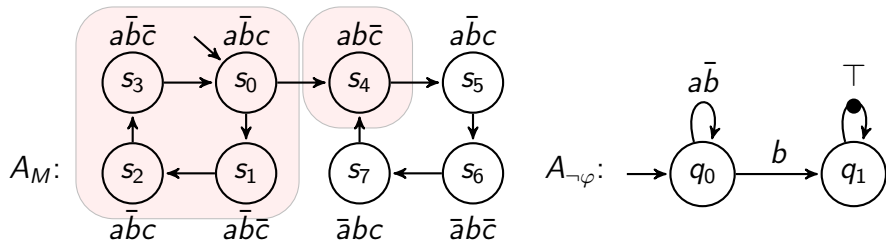
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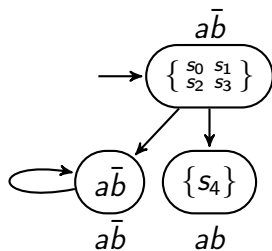
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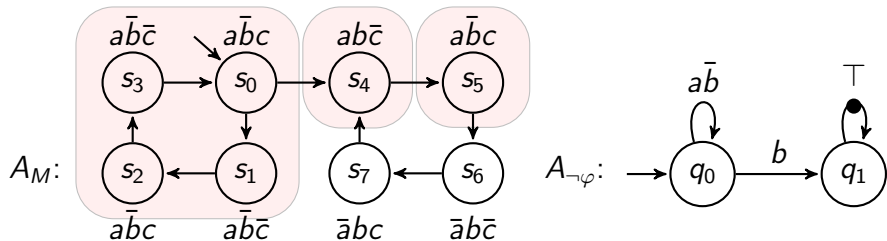
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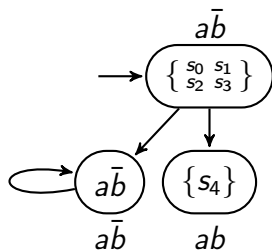
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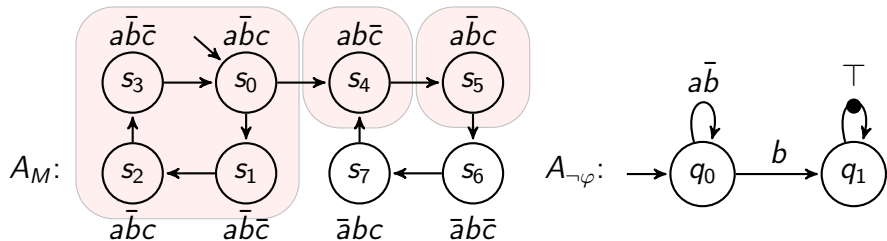
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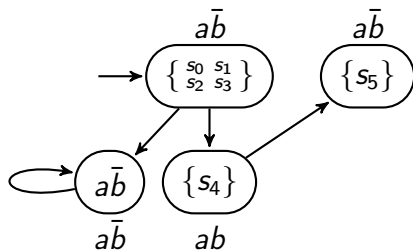
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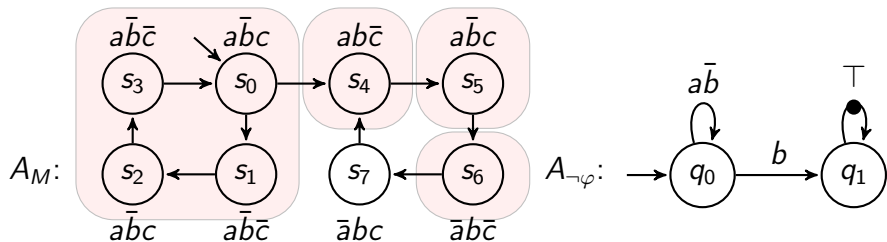
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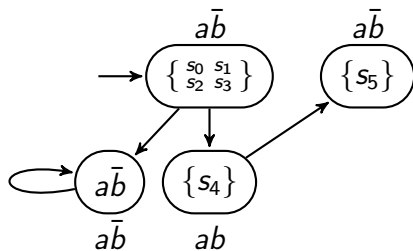
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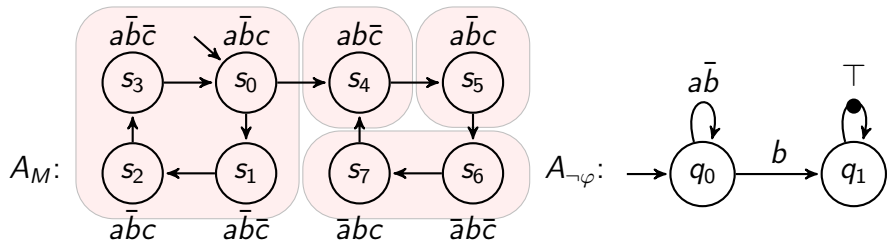
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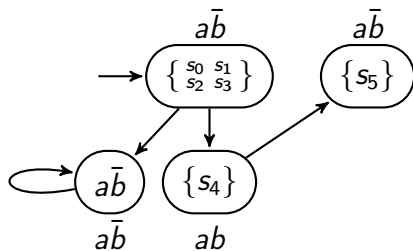
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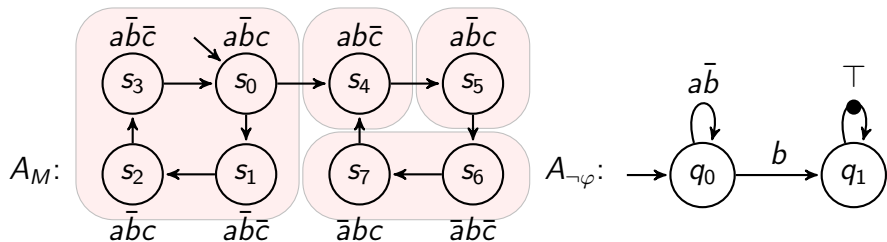
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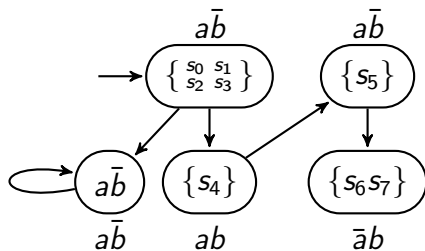
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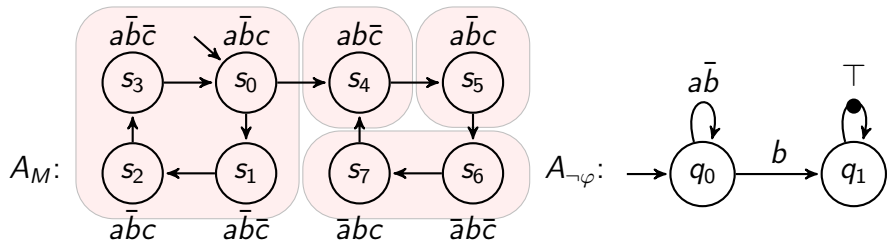
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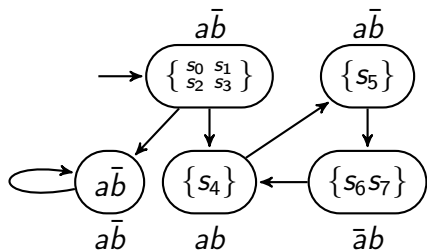
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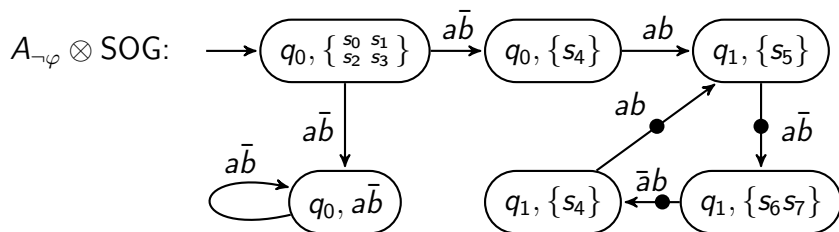
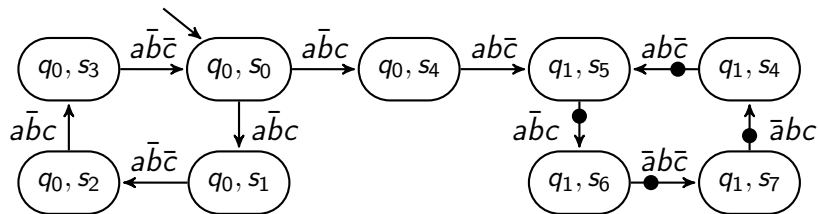


Symbolic Observation Graph:



Product Sizes: Kripke vs. SOG

$A_{\neg\varphi} \otimes A_M$:



Building a Product Directly

High-level
model M

On-the-fly generation
of state-space automaton
 A_M

On-the-fly
synchronized product
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Emptiness check
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) \stackrel{?}{=} \emptyset$

LTL
property φ

LTL
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Negated
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$M \models \varphi$ or
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Building a Product Directly

High-level
model M

Dynamic and on-the-fly generation
of an automaton D such that
 $\mathcal{L}(D) = \emptyset \iff \mathcal{L}(A_{\neg\varphi} \otimes A_M) = \emptyset.$

Emptiness check
 $\mathcal{L}(D) \stackrel{?}{=} \emptyset$

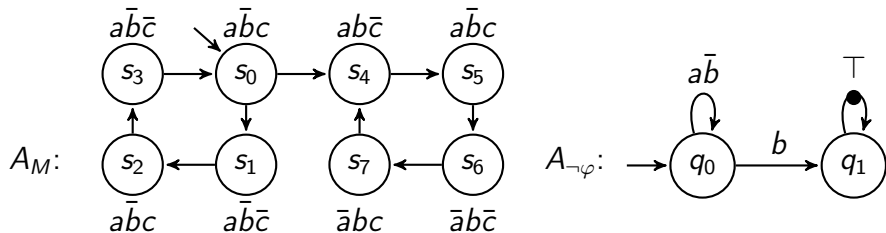
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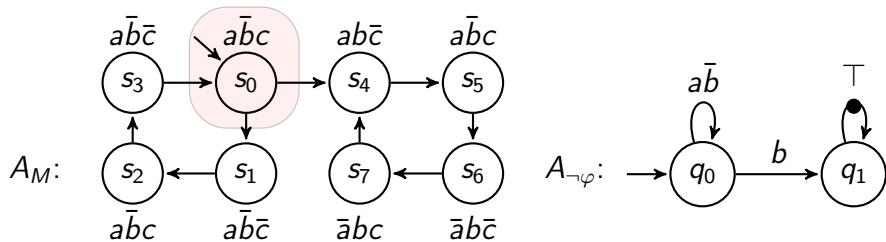
Symbolic Observation Product (SOP)



Symbolic Observation Product:

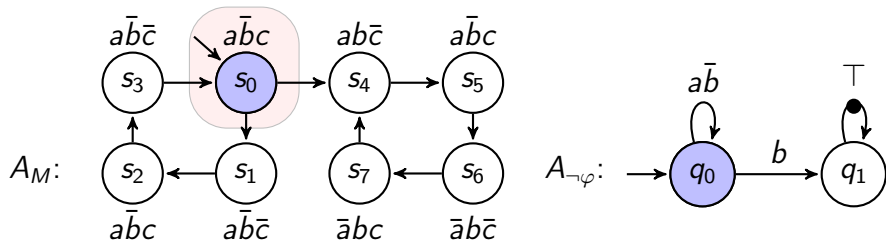
- A dynamic extension of the SOG
- Gather all states whose valuation do not change the propositions observed by the property automaton **at the current point**.

Symbolic Observation Product (SOP)



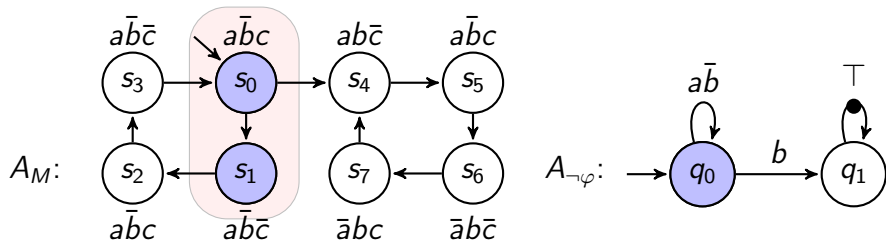
Symbolic Observation **Product**:

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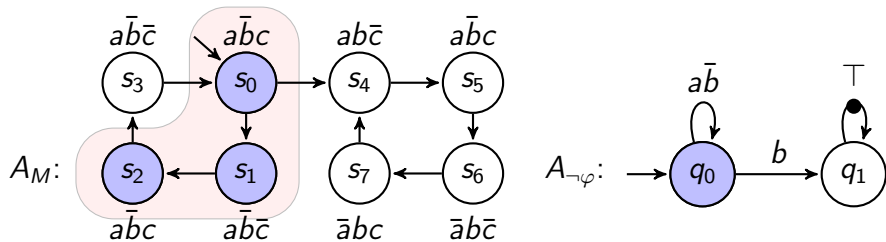
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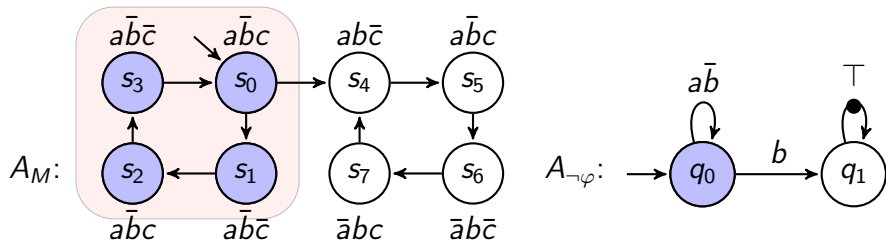
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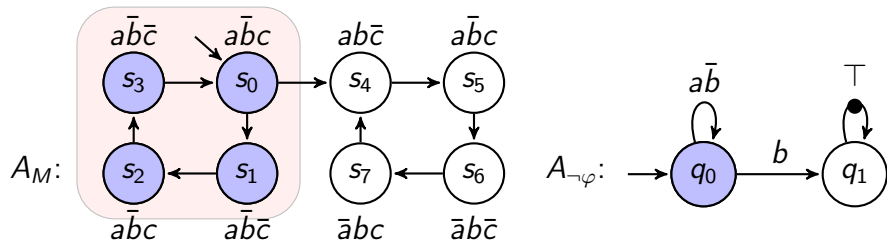
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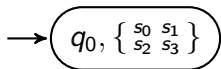


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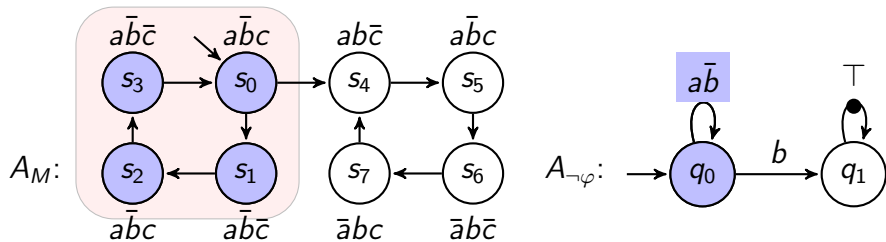
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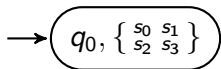
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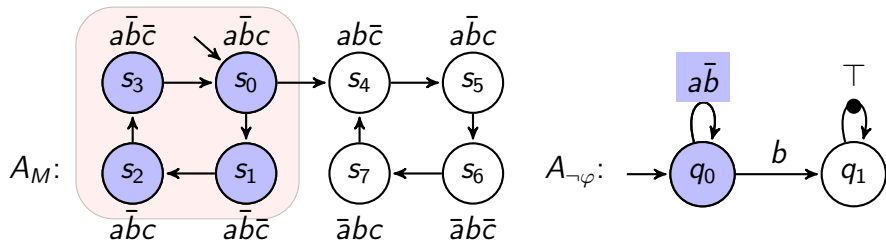
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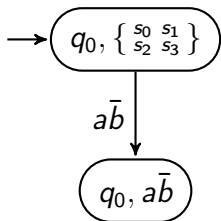
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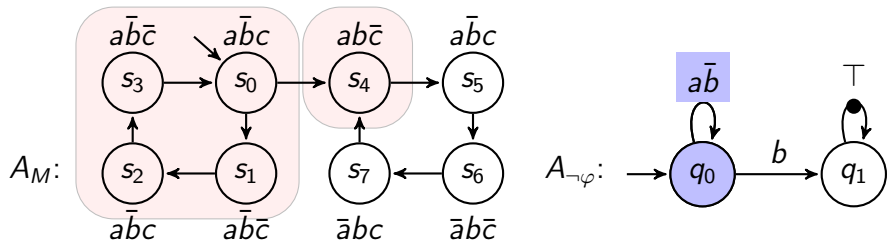
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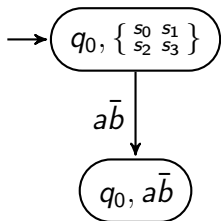
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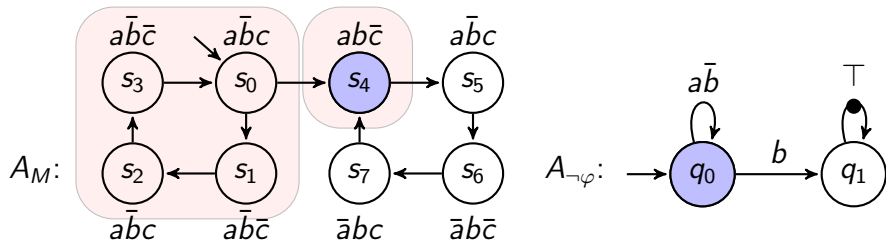
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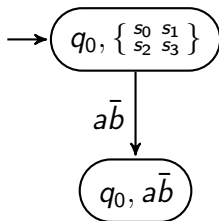
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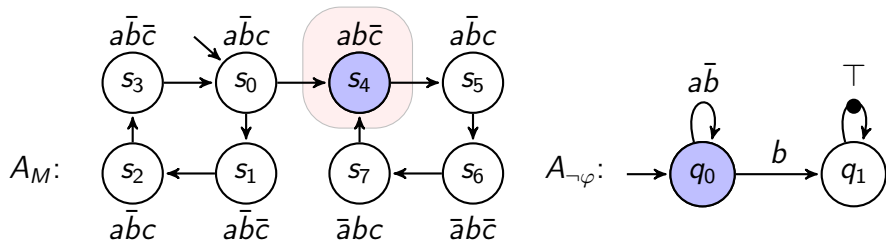
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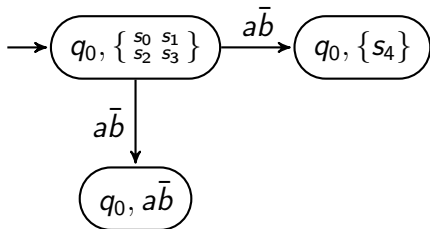
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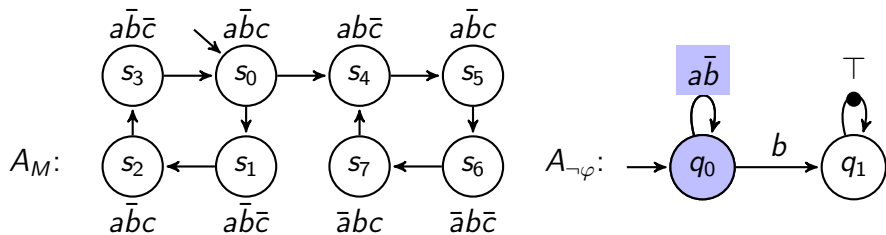
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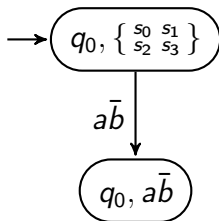
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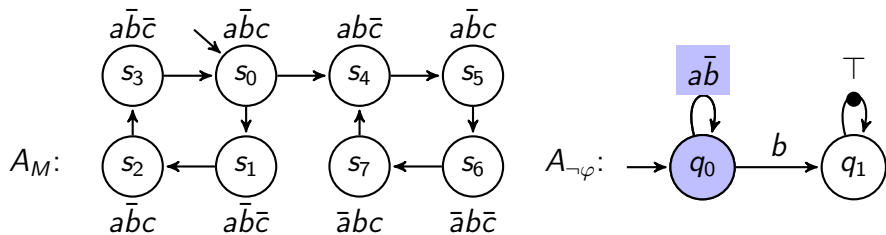
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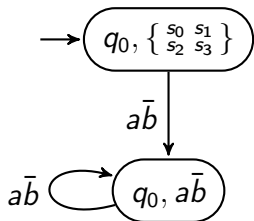
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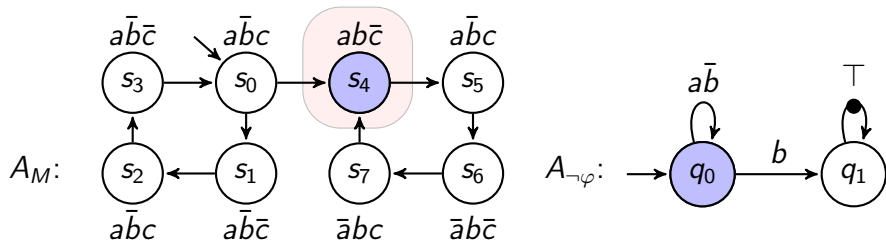
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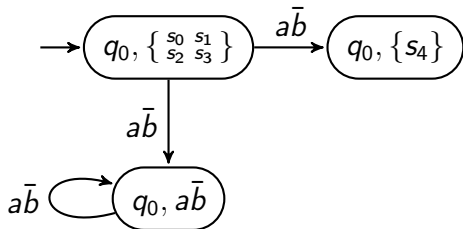
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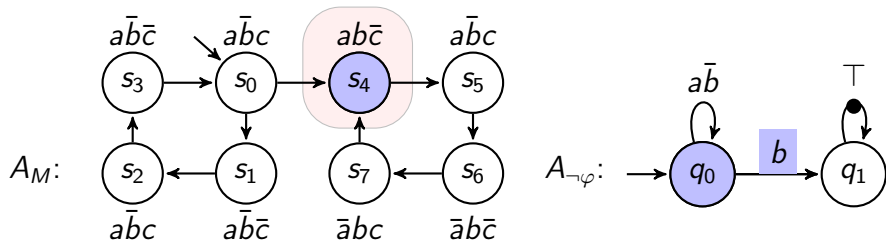
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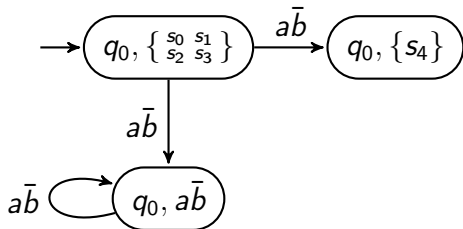
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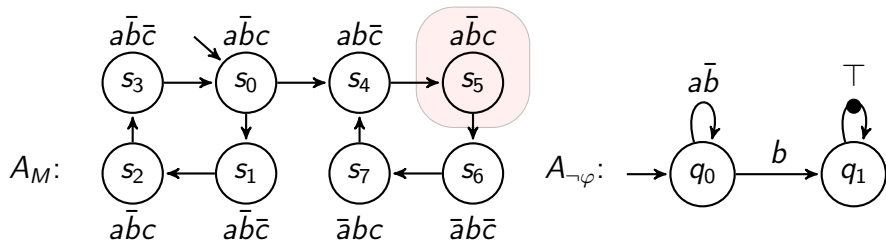
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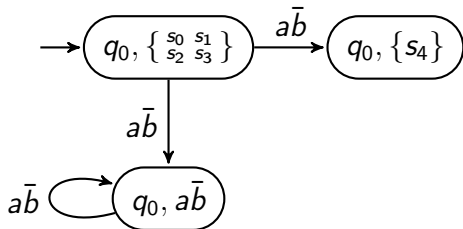
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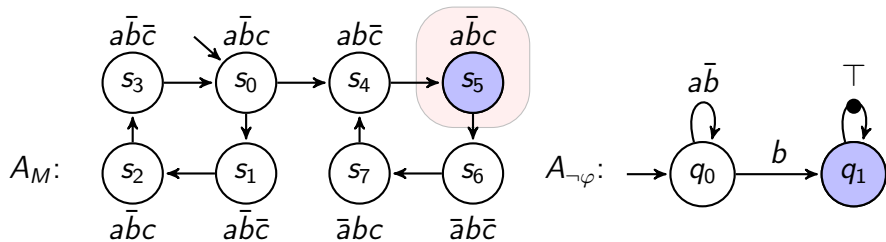
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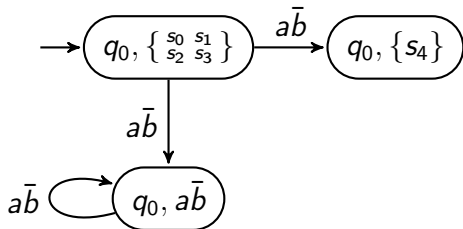
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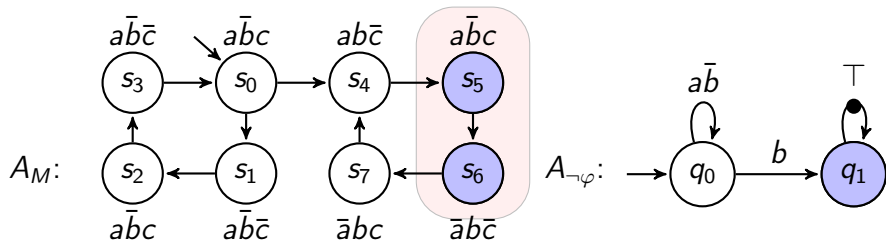
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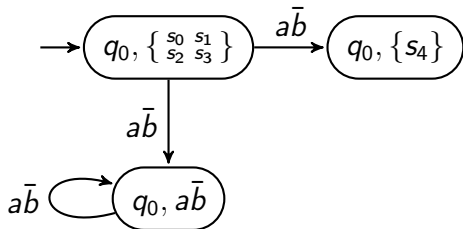
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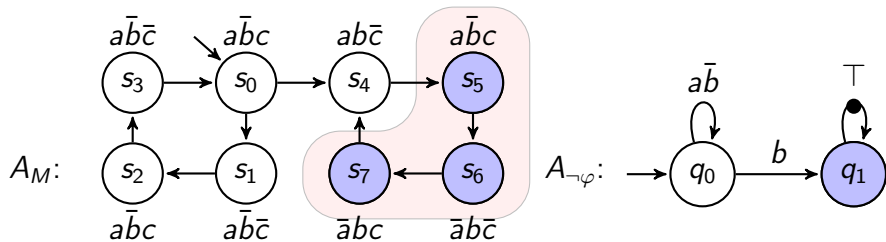
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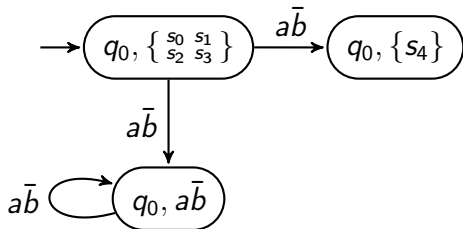
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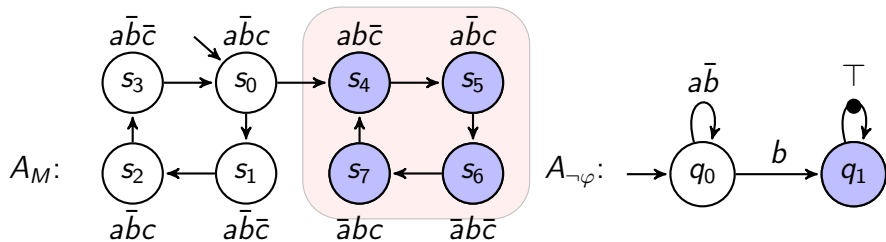
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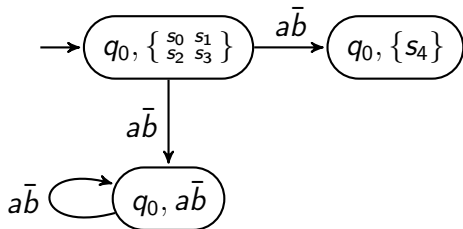
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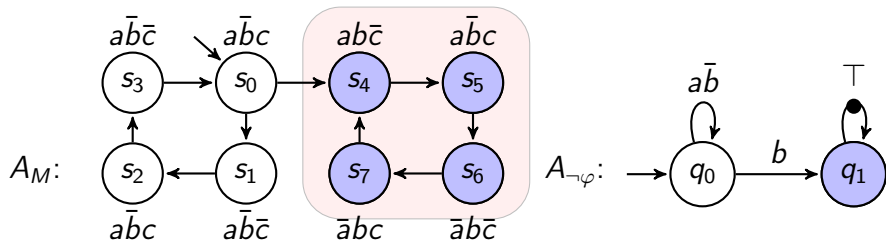
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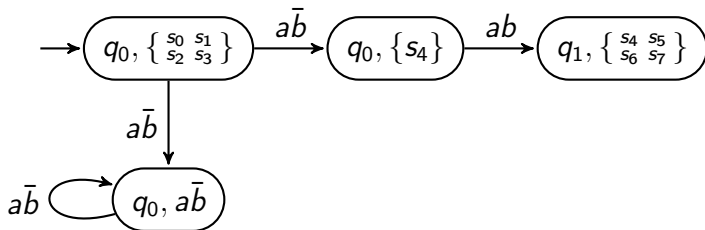
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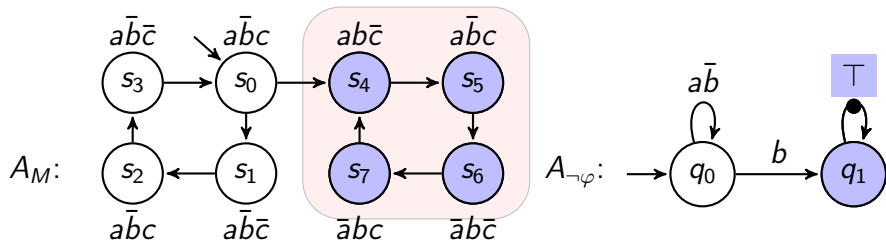
Symbolic Observation Product (SOP)



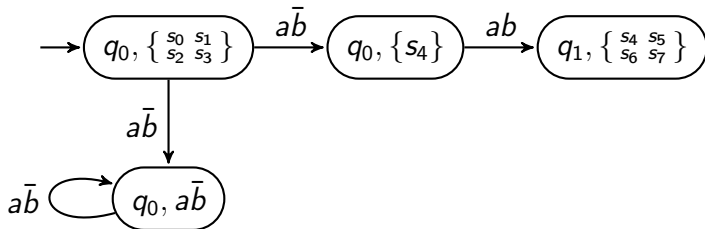
Symbolic Observation Product:



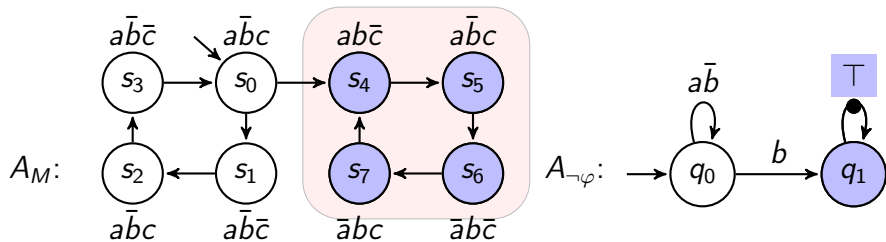
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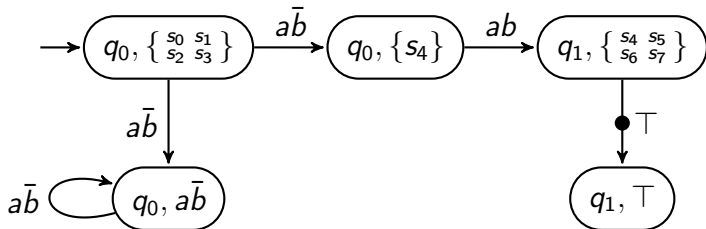
Symbolic Observation Product:



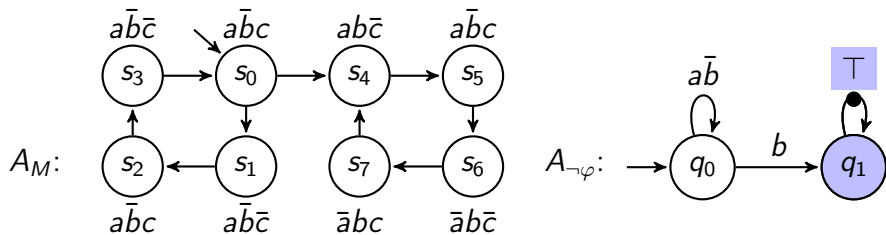
Symbolic Observation Product (SOP)



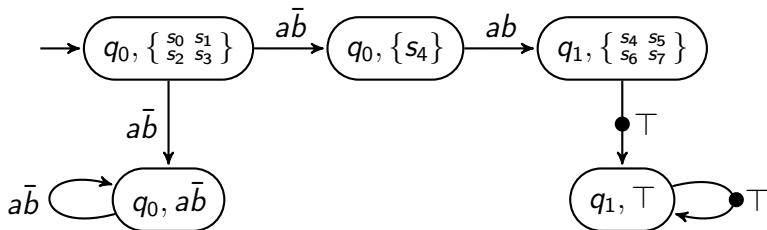
Symbolic Observation Product:




Symbolic Observation Product (SOP)

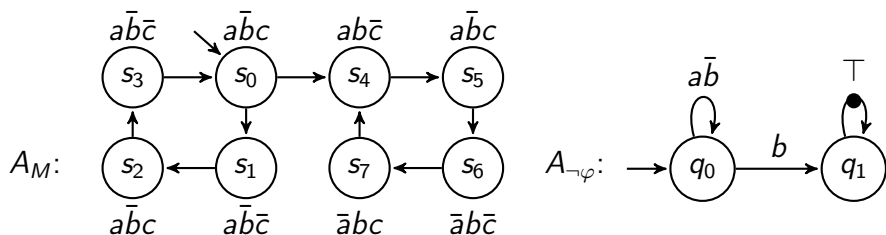


Symbolic Observation Product:



Self-Loop Aggregation Product (SLAP)

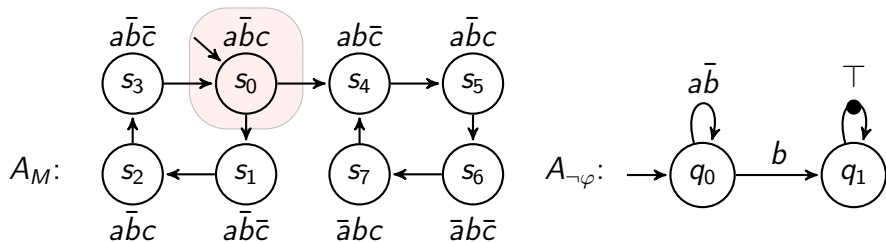
 Self-loop aggregation product - a new hybrid approach to on-the-fly LTL model checking. In ATVA'11



Self-Loop Aggregation Product:

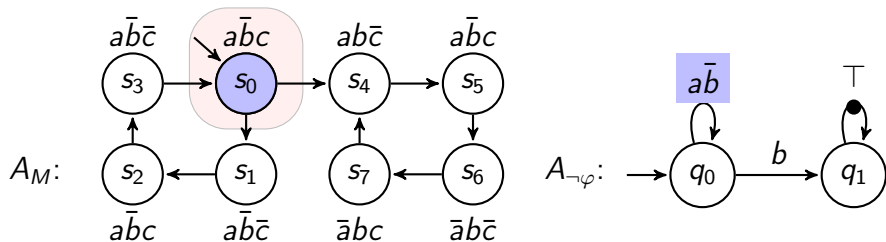
- Specialized synchronized product for full LTL
- Based on the self loops encountered in the property automaton
- Gather all states compatible with the self-loops of the property automaton **at the current point**.

Self-Loop Aggregation Product (SLAP)



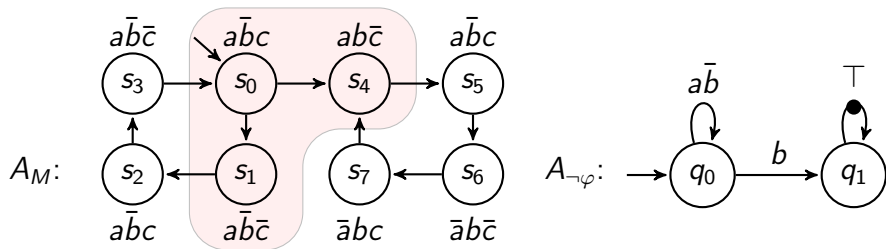
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



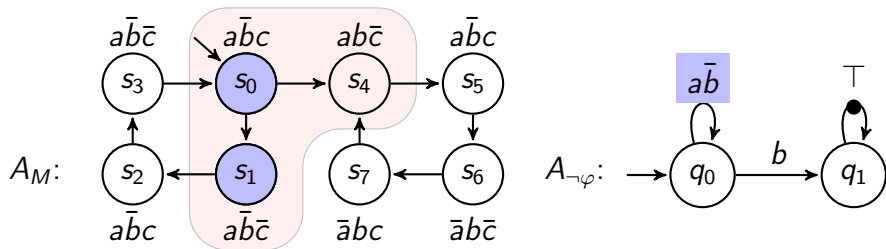
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



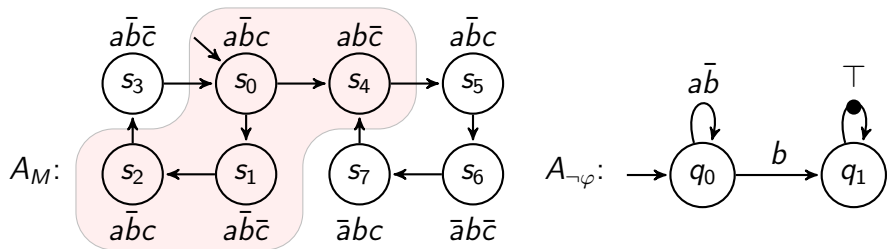
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



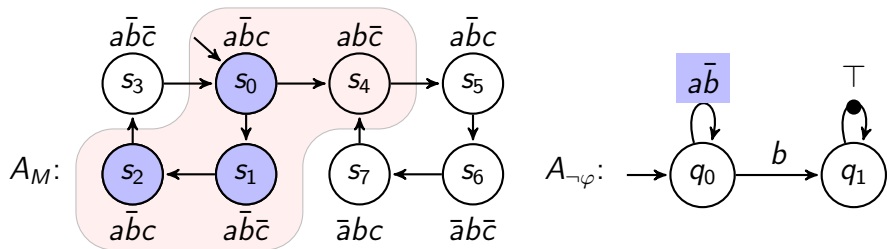
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



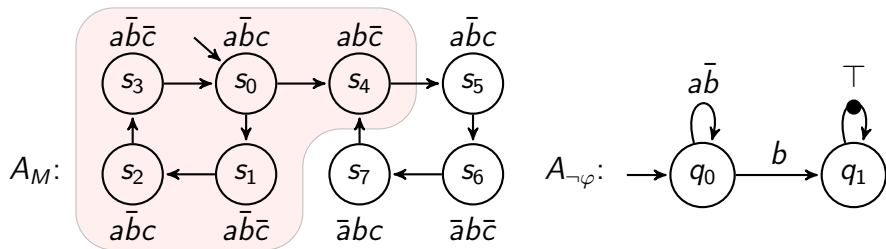
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



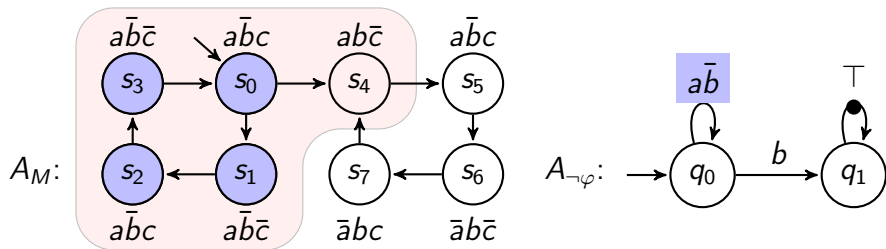
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)



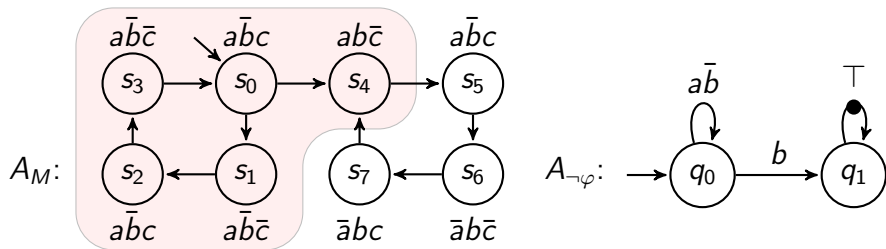
Self-Loop Aggregation **Product**:

Self-Loop Aggregation Product (SLAP)

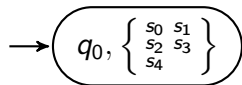


Self-Loop Aggregation **Product**:

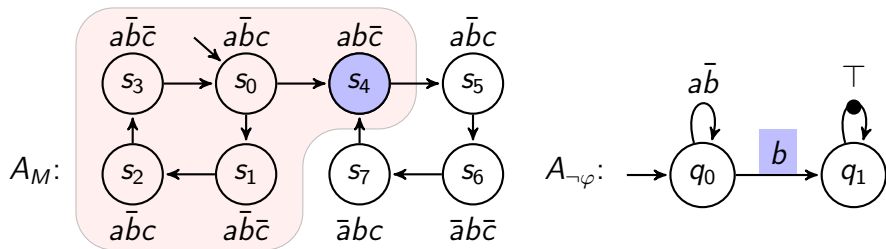
Self-Loop Aggregation Product (SLAP)



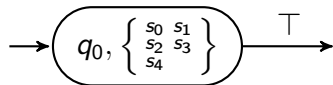
Self-Loop Aggregation Product:



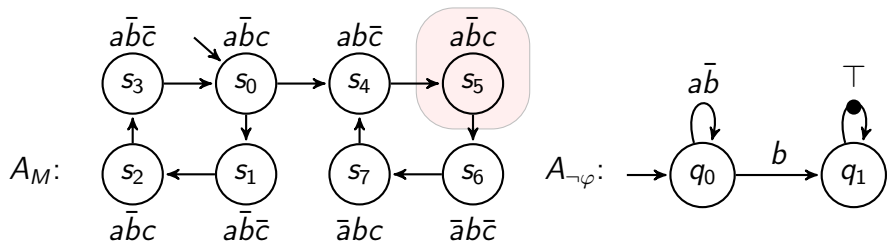
Self-Loop Aggregation Product (SLAP)



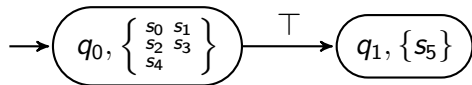
Self-Loop Aggregation Product:



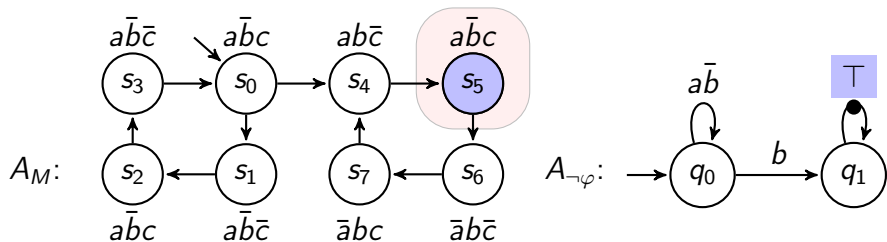
Self-Loop Aggregation Product (SLAP)



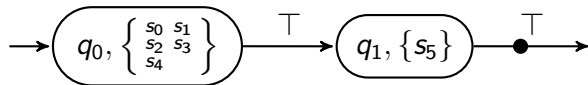
Self-Loop Aggregation Product:



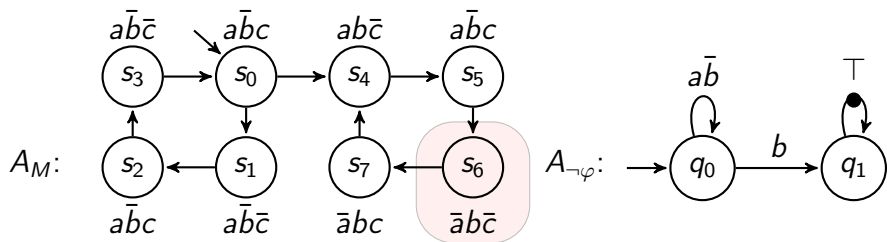
Self-Loop Aggregation Product (SLAP)



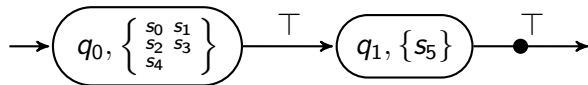
Self-Loop Aggregation Product:



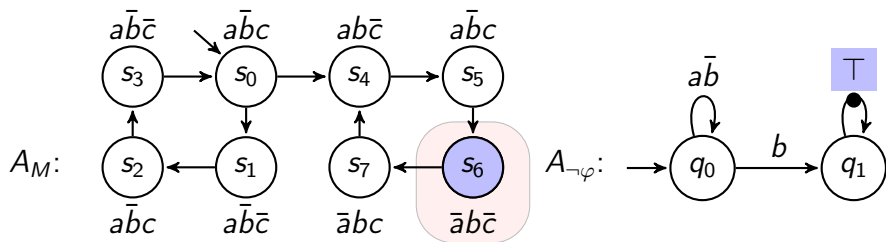
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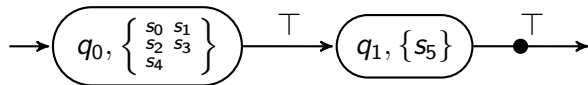
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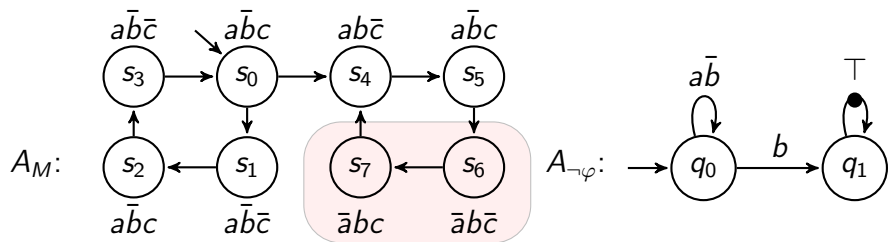
Self-Loop Aggregation Product (SLAP)



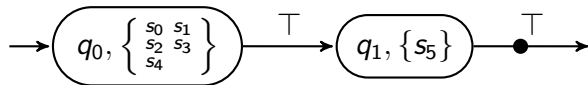
Self-Loop Aggregation Product:



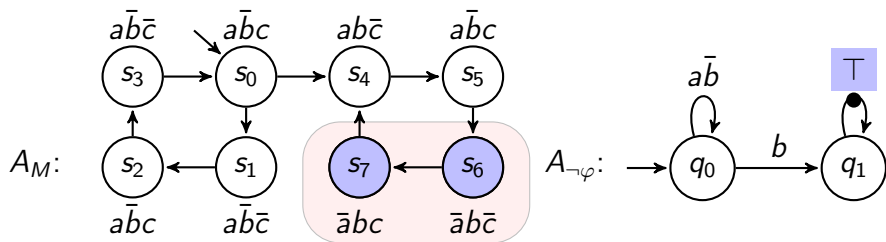
Self-Loop Aggregation Product (SLAP)



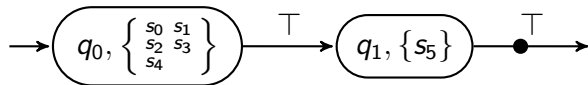
Self-Loop Aggregation Product:



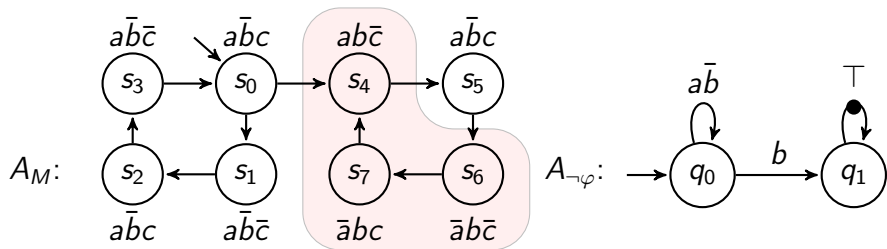
Self-Loop Aggregation Product (SLAP)



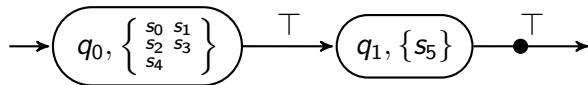
Self-Loop Aggregation Product:



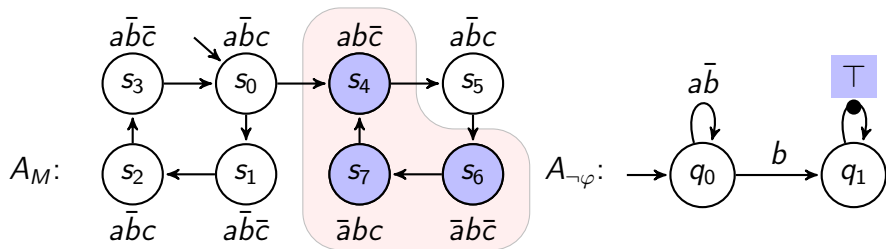
Self-Loop Aggregation Product (SLAP)



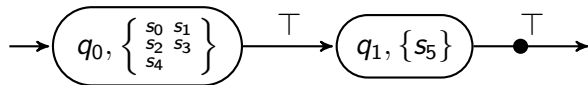
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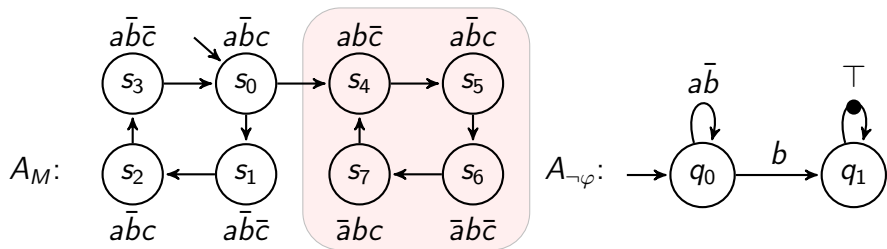
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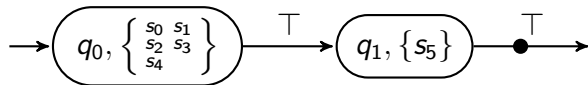
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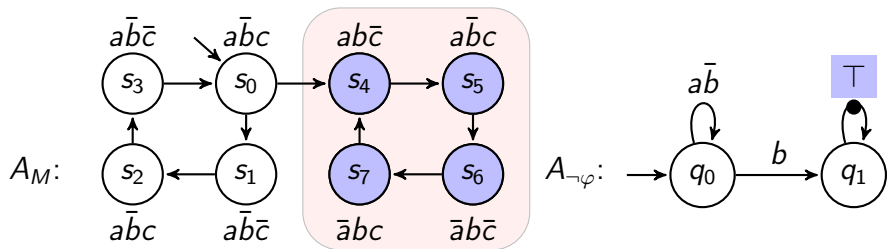
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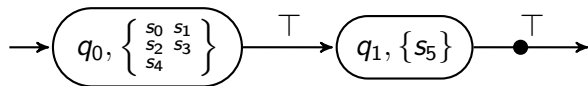
Self-Loop Aggregation Product:



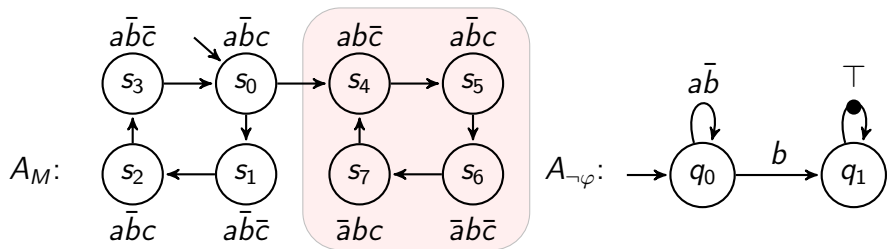
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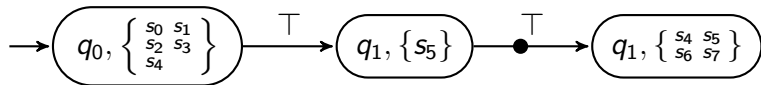
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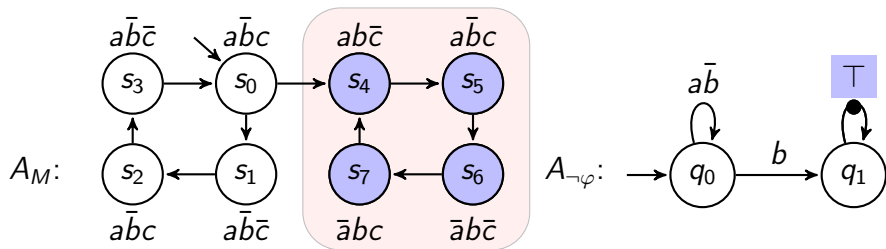
Self-Loop Aggregation Product (SLAP)



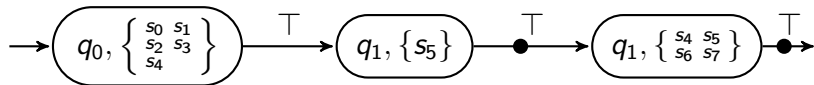
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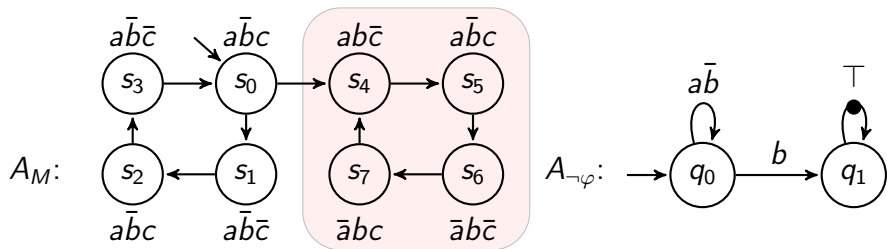
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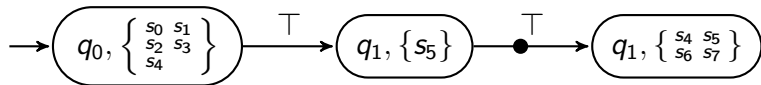
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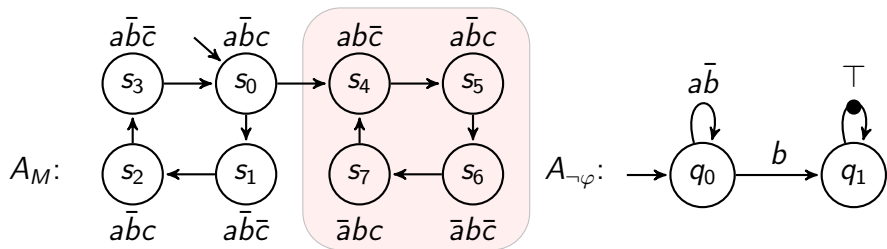
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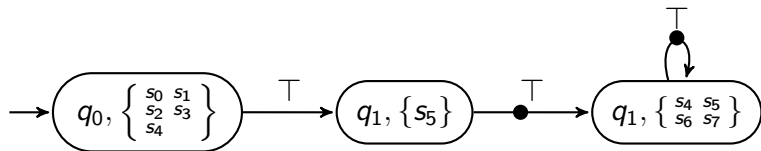
Self-Loop Aggregation Product:



Self-Loop Aggregation Product (SLAP)

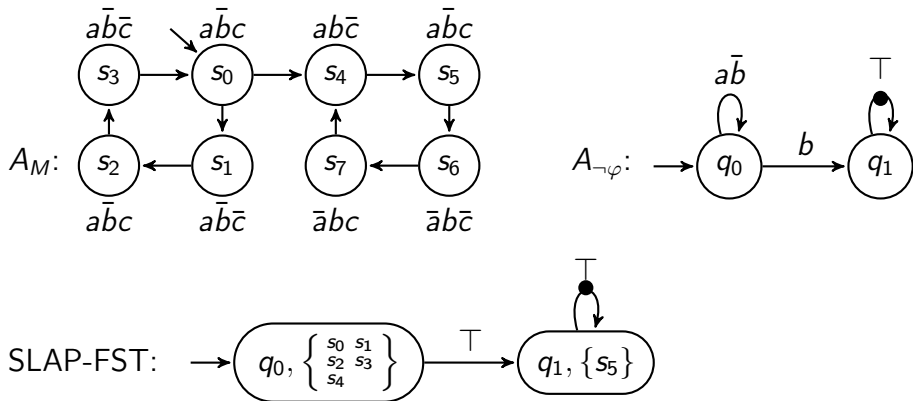


Self-Loop Aggregation Product:



SLAP-FST

- FST = Fully Symbolic search in Terminal automata states
- When reaching a state of $A_{\neg\varphi}$ which is terminal (and accepting) use a fully symbolic search.



Summary of methods

	Expl	BCZ	SOG	SOP	SLAP	PDP	FS
logic	LTL		LTL \ X		LTL		
prop. aut.	expl						symb/expl
data str.	graph	BDD+graph				BDD[]	BDD
empt. chk	expl					symb (OWCTY/EL)	

Experimental Framework

Implemented algorithms

BCZ, SOG, SOP, SLAP, SLAP-FST, EL, OWCTY

Tools used

SDD Hierarchical Set Decision Diagrams (<http://ddd.lip6.fr/>)

- Hierarchy (memory gain)
- Automatic saturation (improves fixpoint computations)

Spot Model checking library (<http://spot.lip6.fr/>)

- Good LTL-to-TGBA translation
- Explicit emptiness-check algorithms

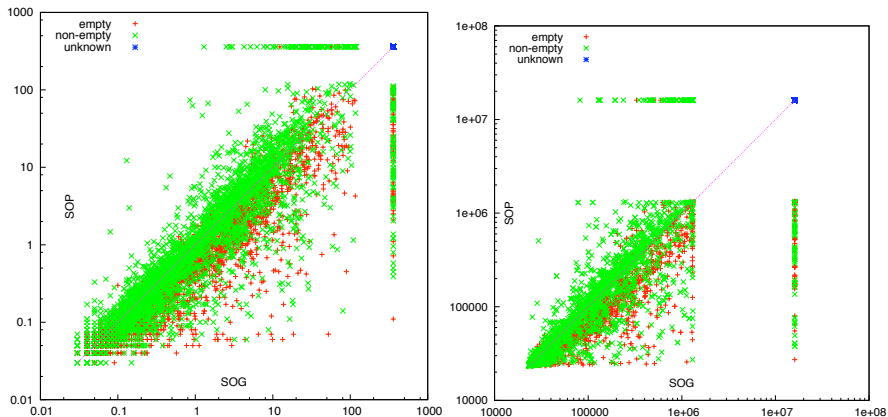
Data

Models Scalable toy models with $10^6 \dots 10^{66}$ states.

Formulas Random formulas (filtered), plus a set of (random) weak fairness properties.

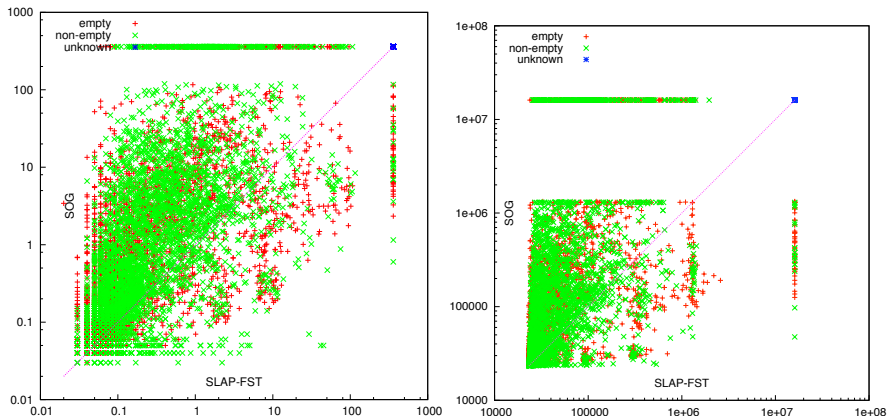
Scatter Plots: Stuttering invariant algorithms SOG vs SOP

Runtime in seconds. Timeout at 120s. Memory in Kilobytes.
Garbage collection threshold at 1.3 GB



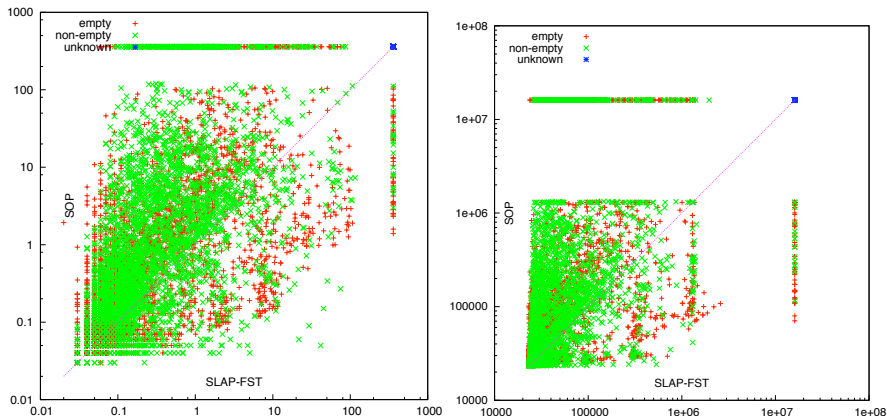
Scatter Plots: SLAP-FST vs SOG

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Garbage collection threshold at 1.3 GB



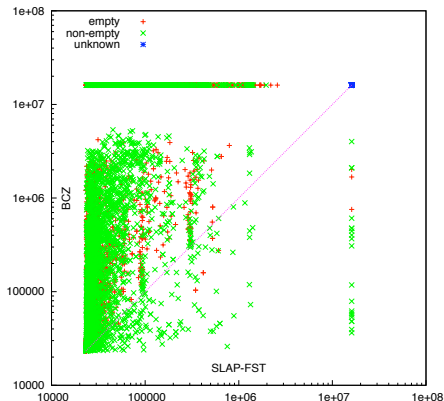
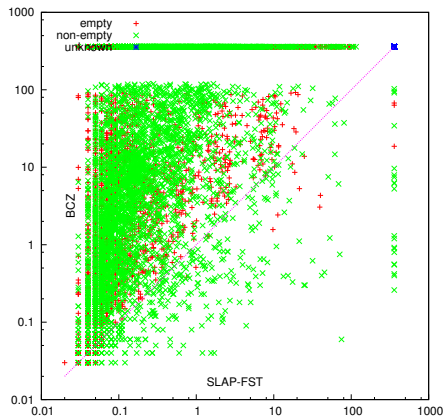
Scatter Plots: SLAP-FST vs SOP

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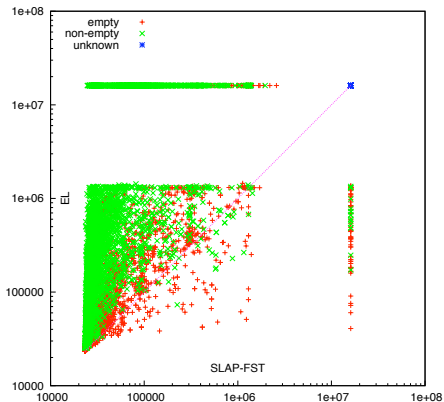
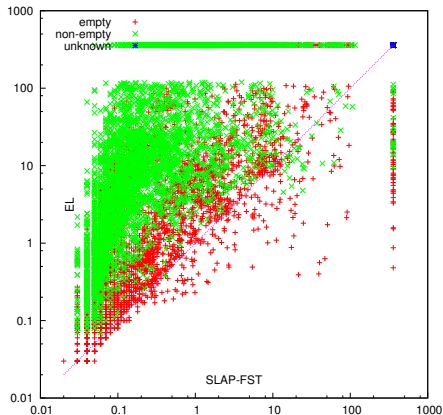
Scatter Plots: SLAP-FST vs. BCZ

Runtime in seconds. Timeout at 120s. Memory in Kilobytes.
Garbage collection threshold at 1.3 GB



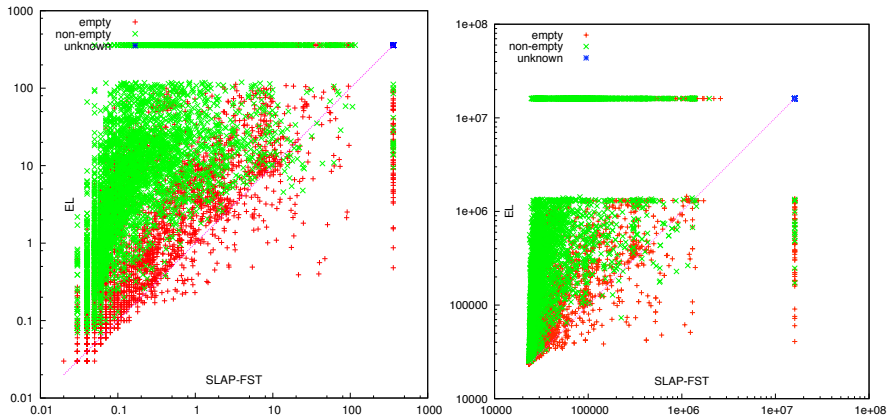
Scatter Plots: SLAP-FST vs. Fully Symbolic

Runtime in seconds. Timeout at 120s. Memory in Kilobytes.
Garbage collection threshold at 1.3 GB



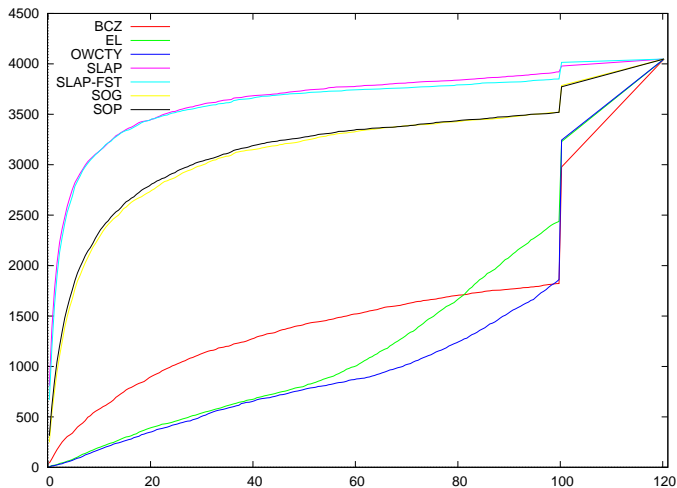
Scatter Plots: SLAP-FST vs. Fully Symbolic

Runtime in seconds. Timeout at 120s. Memory in Kilobytes.
Garbage collection threshold at 1.3 GB

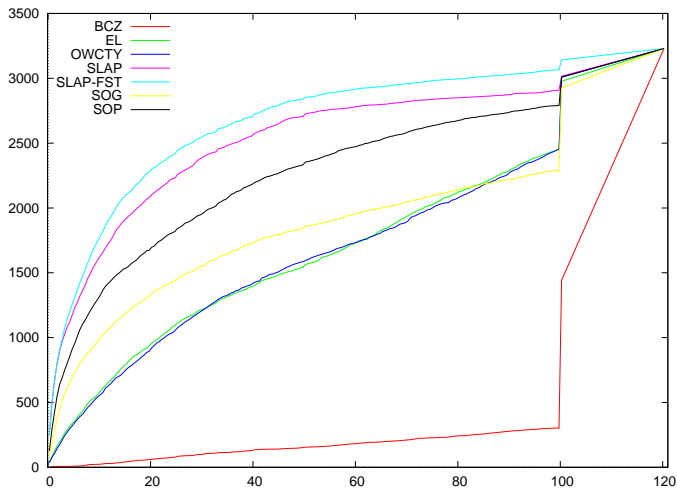


http://move.lip6.fr/software/DDD/ltl_bench.html

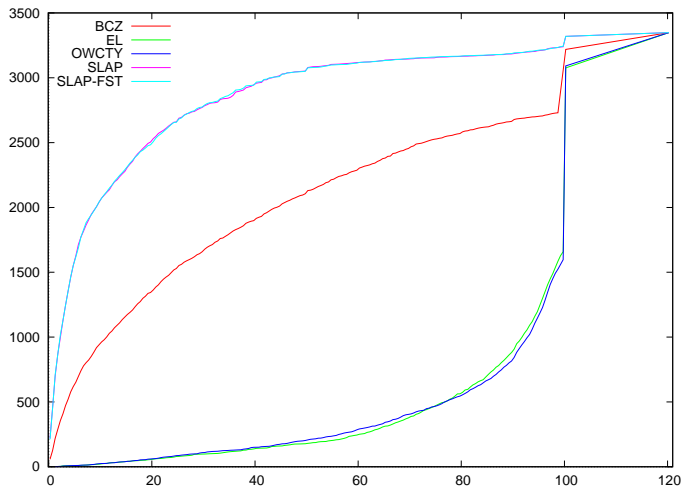
Cumulative Plot: Violated Stuttering Formulas



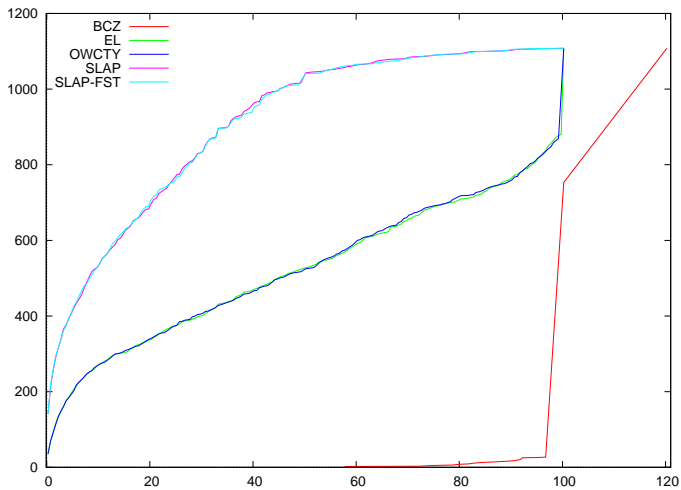
Cumulative Plot: Verified Stuttering Formulas



Cumulative Plot: Violated Non Stuttering Formulas



Cumulative Plot: Verified Non Stuttering Formulas



Conclusion and Perspectives

- Two new hybrid approaches: Aggregates states at the product level, not in the Kripke structure
 - SOP
 - SLAP and SLAP-FST
- SLAP-FST most competitive algorithm of those we compared, **on this benchmark**

Conclusion and Perspectives

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- Compare our hybrid approaches to PDP
- Partial order
- Testing automata
- SOPED SOG and SLAPED SOG