Deadlock Prevention for Flexible Manufacturing Systems

A Petri Net Approach

ZhiWu Li

Systems Control and Automation Group
School of Electro-Mechanical Engineering
Xidian University
No.2 South Taibai Road
Xian 710071, China
email: zhwli@xidian.edu.cn

January 22, 2010

Outline

Outline

A Small Manufacturing Example

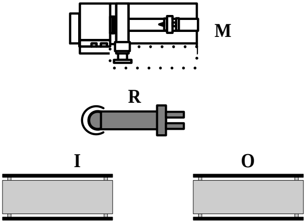


Figure: A small manufacturing system.

Example

- The system can produce a kind of parts.
- The robot is used to upload and download the machine.
- Resource requirement sequence: robot→machine→robot.

A Small Manufacturing Example

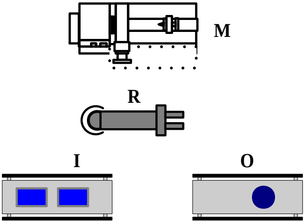


Figure: A small manufacturing system.

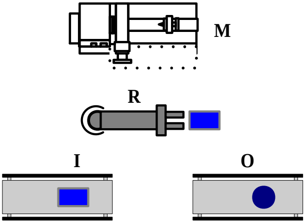


Figure: A small manufacturing system.

A Small Manufacturing Example

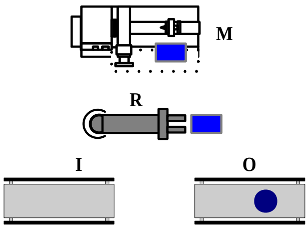


Figure: A small manufacturing system.

Example

- A deadlock occurs when the machine is processing a part and the robot picks up a raw material from input buffer.
- Deadlock Resolution: Robot is not allowed to pick up a raw material if the machine is occupied.

A Small Manufacturing Example

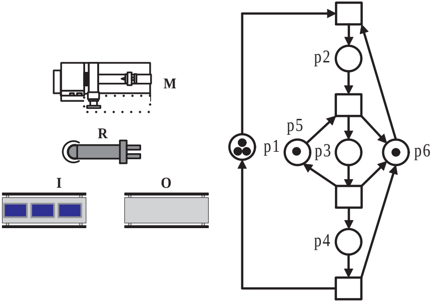


Figure: A small manufacturing system.

Example

- p_1 – input/output buffers
- p_5 – machine
- p_6 – robot
- p_2 – uploading
- p_3 – processing
- p_4 – downloading

A Small Manufacturing Example

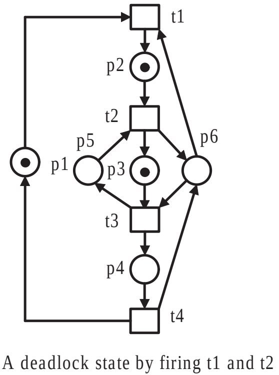


Figure: A small manufacturing system.

Example

- A deadlock occurs when
- robot picks up a raw material
- machine is processing a part

A Small Manufacturing Example

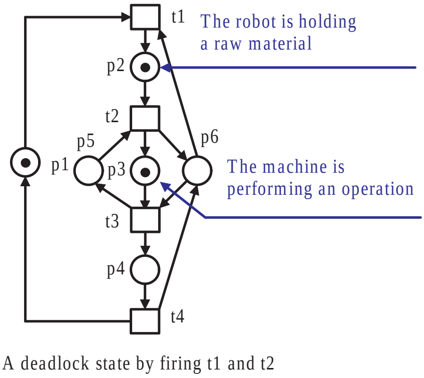


Figure: A small manufacturing system.

Example

- A deadlock occurs when
- robot picks up a raw material
- machine is processing a part

A small manufacturing Example

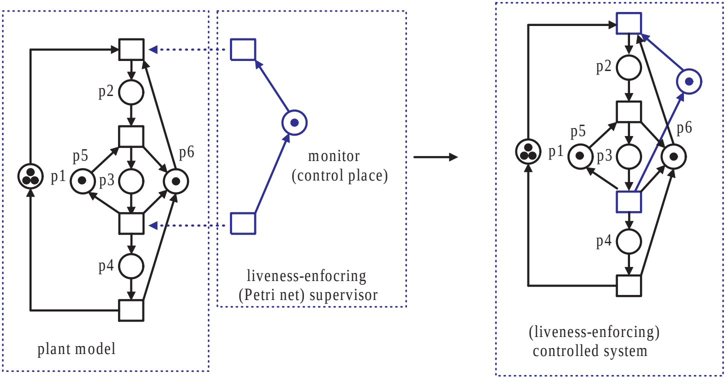


Figure: Plant, supervisor, and controlled system.

Deadlock Conditions in Resource Allocation Systems

- Mutual exclusion: activities exclusively hold every resource they have acquired
- Hold and wait: activities hold resources while waiting for additional resources to proceed
- No preemption: resources cannot be removed while it is being used
- Circular wait: there must exist a circular chain of processes.

The first three conditions are determined by the physical characteristics of a system and its resources, i.e., they do not change with time.

Outline

Deadlock Handling Formalisms

- Graph Theory (Digraphs)
- Automata
- Petri Nets

Although there is no inclusive arguments making evident the superiority of one of these methodologies to another, Petri nets are increasingly becoming a full-fledged mathematical tool to investigate the deadlock problems in flexible manufacturing.

Deadlock Control Strategy

- Deadlock Detection and Recovery
- Deadlock Avoidance
- Deadlock Prevention

Deadlock Detection and Recovery

- A deadlock detection and recovery approach permits the occurrence of deadlocks. When a deadlock occurs, it is detected and then the system is put back to a deadlock-free state, by simply reallocating the resources.
- The efficiency of this approach depends upon the response time of the implemented algorithms for deadlock detection and recovery.
- In general, these algorithms require a large amount of data and may become complex when several types of shared resources are considered.

Deadlock Avoidance

- In deadlock avoidance, at each system state an on-line control policy is used to make a correct decision to proceed among the feasible evolutions.
- The main purpose of this approach is to keep the system away from deadlock states.
- Aggressive methods usually lead to higher resource utilization and throughput, but do not totally eliminate all deadlocks for some cases. In such cases if a deadlock arises, suitable recovery strategies are still required.
- Conservative methods eliminate all unsafe states and deadlocks, and often some good states, thereby degrading the system performance. On the other hand, they are intended to be easy to implement.

Deadlock Prevention

- Deadlock prevention is usually achieved by using an off-line computational mechanism to control the request for resources to ensure that deadlocks never occur.
- It imposes constraints on a system to prevent it from reaching deadlock states. In this case, the computation is carried out off-line in a static way and once the control policy is established, the system can no longer reach undesirable deadlock states.
- They require no run-time cost since problems are solved in system design and planning stages.
- The major criticism is that they tend to be too conservative, thereby reducing the resource utilization and system productivity.

Deadlock Control via Siphons

- Deadlock arises from token depletion in certain PN structural objects called siphons.
- Controlling them such that they are always marked can prevent deadlock from occurrence.

Plant Petri Net Model + Liveness-enforcing Petri net Supervisor

=

Controlled System

Formal Definitions

Definition

A generalized Petri net structure is $N = (P, T, F, W)$, P and T are finite, non-empty, and disjoint. P is a set of places; T is a set of transitions with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$; $F \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net; $W : (P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ assigns a weight to an arc by defining $W(x, y) > 0$ if $(x, y) \in F$, and $W(x, y) = 0$ otherwise.

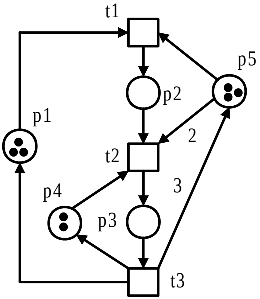


Figure: A Petri net (N, M_0).

Formal Definitions

Definition

- A marking M of a Petri net N is a mapping from P to $\mathbb{N}$.
- $M(p)$ denotes the number of tokens in place p. A place p is marked by a marking M iff $M(p) > 0$.
- A subset $S \subseteq P$ is marked by M iff at least one place in S is marked by M.
- The sum of tokens of all places in S is denoted by $M(S)$, i.e., $M(S) = \sum_{p \in S} M(p)$.
- S is said to be empty at M iff $M(S) = 0$.
- (N, M_0) is called a net system or marked net and M_0 is called an initial marking of N.

Formal Definitions

Example

The figure shows a Petri net with
$P = \{p_1, p_2, p_3, p_4, p_5\}$,
$T = \{t_1, t_2, t_3\}$,
$F = \{(p_1, t_1), (t_3, p_1), (p_2, t_2), (t_1, p_2), (p_3, t_3), (t_2, p_3), (p_4, t_2), (t_3, p_4), (p_5, t_1), (p_5, t_2), (t_3, p_5)\}$,
$W(p_1, t_1) = W(t_3, p_1) = W(p_2, t_2) = W(t_1, p_2) = W(p_3, t_3) = W(t_2, p_3) = W(p_4, t_2) = W(t_3, p_4) = W(p_5, t_1) = 1$, $W(p_5, t_2) = 2$, and $W(t_3, p_5) = 3$.

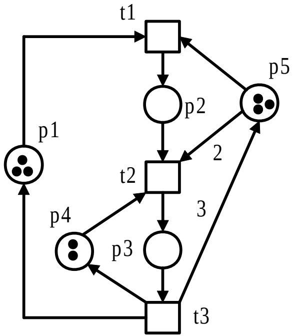


Figure: A Petri net (N, M_0).

Formal Definitions

Example

Either of places p_1 and p_5 has three tokens, denoted by three black dots inside. Place p_4 holds two tokens and there is no token in p_2 and p_3. This token distribution leads to the initial marking of the net with $M_0 = 3p_1 + 2p_4 + 3p_5$.

Formal Definitions

Definition

Let $x \in P \cup T$ *be a node of net* $N = (P, T, F, W)$. *The preset of* x *is defined as* ${}^{\bullet}x = \{y \in P \cup T | (y, x) \in F\}$. *While the postset of* x *is defined as* $x^{\bullet} = \{y \in P \cup T | (x, y) \in F\}$. *This notation can be extended to a set of nodes as follows: given* $X \subseteq P \cup T$, ${}^{\bullet}X = \cup_{x \in X} {}^{\bullet}x$, *and* $X^{\bullet} = \cup_{x \in X} x^{\bullet}$. *Given place p, we denote* $max\{W(p, t) \mid t \in p^{\bullet}\}$ *by* $max_{p^{\bullet}}$.

Formal Definitions

Example

We have ${}^\bullet t_1 = \{p_1, p_5\}$, ${}^\bullet t_2 = \{p_2, p_4, p_5\}$, $t_2^\bullet = \{p_3\}$, $t_3^\bullet = \{p_1, p_4, p_5\}$, ${}^\bullet p_3 = \{t_2\}$, $p_3^\bullet = \{t_3\}$, ${}^\bullet p_5 = \{t_3\}$, and $p_5^\bullet = \{t_1, t_2\}$. Let $S = \{p_3, p_5\}$. Then, ${}^\bullet S = {}^\bullet p_3 \cup {}^\bullet p_5 = \{t_2, t_3\}$ and $S^\bullet = p_3^\bullet \cup p_5^\bullet = \{t_1, t_2, t_3\}$. It is easy to see that $max_{p_5^\bullet} = 2$ and $\forall p \neq p_5$, $max_{p^\bullet} = 1$. $\diamond$

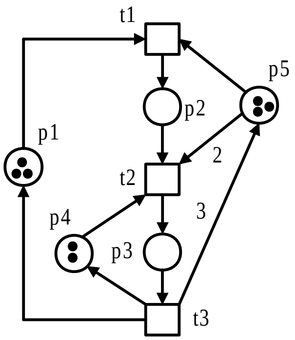


Figure: A Petri net (N, M_0).

Formal Definitions

Definition

A transition $t \in T$ is enabled at a marking M iff $\forall p \in {}^\bullet t$, $M(p) \geq W(p, t)$. This fact is denoted as $M[t\rangle$. Firing it yields a new marking M' such that $\forall p \in P$, $M'(p) = M(p) - W(p, t) + W(t, p)$, as denoted by $M[t\rangle M'$. M' is called immediately reachable from M. Marking M'' is said to be reachable from M if there exists a sequence of transitions $\sigma = t_0 t_1 \cdots t_n$ and markings $M_1, M_2, \cdots,$ and M_n such that $M[t_0\rangle M_1[t_1\rangle M_2 \cdots M_n[t_n\rangle M''$ holds. The set of markings reachable from M in N is called the reachability set of Petri net (N, M) and denoted as $R(N, M)$.

Formal Definitions

Example

t_1 is enabled at $M_0 = 3p_1 + 2p_4 + 3p_5$: $M_0(p_1) = 3 > W(p_1, t_1) = 1$, and $M_0(p_5) = 3 > W(p_5, t_1) = 1$. Firing t_1 leads to M_1 with $M_1(p_1) = M_0(p_1) - W(p_1, t_1) + W(t_1, p_1) = 2$, $M_1(p_2) = M_0(p_2) - W(p_2, t_1) + W(t_1, p_2) = 1$, $M_1(p_3) = 0$, $M_1(p_4) = 2$, and $M_1(p_5) = 2$.

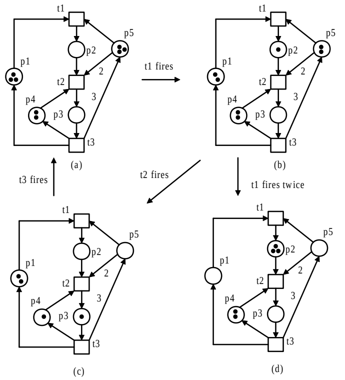


Figure: The evolution of a PN (N, M_0).

Formal Definitions

Example

In marking M_1, both t_1 and t_2 are enabled. Fig. **??**(c) is the net after t_2 fires from M_1 and Fig. **??**(d) is the net after transition sequence $\sigma = t_1 t_1$ fires from M_1, i.e, t_1 fires twice. As a result, the reachability set of the net in Fig. **??**(a) is $R(N, M_0) = \{M_0, M_1, M_2, M_3, M_4\}$, where $M_0 = 3p_1 + 2p_4 + 3p_5$, $M_1 = 2p_1 + p_2 + 2p_4 + 2p_5$, $M_2 = 2p_1 + p_3 + p_4$, $M_3 = p_1 + 2p_2 + 2p_4 + p_5$, and $M_4 = 3p_2 + 2p_4$. ◇

Formal Definitions

Definition

A net $N = (P, T, F, W)$ *is pure (self-loop free) if* $\forall x, y \in P \cup T$, $W(x, y) > 0$ *implies* $W(y, x) = 0$.

Definition

A pure net $N = (P, T, F, W)$ *can be represented by its incidence matrix* $[N]$, *where* $[N]$ *is a* $|P| \times |T|$ *integer matrix with* $[N](p, t) = W(t, p) - W(p, t)$. *For a place* p *(transition* t*), its incidence vector is denoted by* $[N](p, \cdot)$ *(*$[N](\cdot, t)$*).*

Formal Definitions

Example

The incidence matrix of the net in figure is shown below.

$$[N] = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & -2 & 3 \end{pmatrix}$$

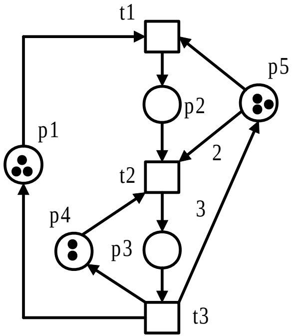


Figure: A Petri net (N, M_0).

Formal Definitions

The incidence matrix $[N]$ of a net N can be naturally divided into two parts $[N]^+$ and $[N]^-$ according to the token flow by defining $[N] = [N]^+ - [N]^-$, where $[N]^+(p, t) = W(t, p)$ and $[N]^-(p, t) = W(p, t)$ that are called input (incidence) matrix and output (incidence) matrix, respectively.

Formal Definitions

Example

$$[N]^+ = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix},$$

$$[N]^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$$

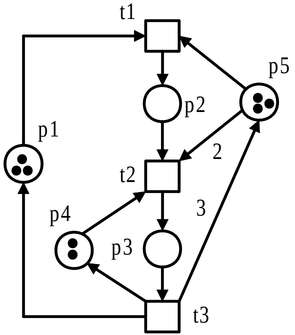


Figure: A Petri net (N, M_0).

Formal Definitions

Definition

Given a Petri net (N, M_0), $t \in T$ *is live under* M_0 *iff* $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$. (N, M_0) *is live iff* $\forall t \in T$, t *is live under* M_0. *It is dead under* M_0 *iff* $\nexists t \in T$, $M_0[t\rangle$. *It is deadlock-free (weakly live or live-lock) iff* $\forall M \in R(N, M_0)$, $\exists t \in T$, $M[t\rangle$.

Definition

Petri net (N, M_0) *is quasi-live iff* $\forall t \in T$, *there exists* $M \in R(N, M_0)$ *such that* $M[t\rangle$.

Formal Definitions

Example

The net shown in Fig.(a) is deadlock-free since transitions t_1 and t_2 are live. While the net in Fig.(b) is live since all transitions are live. ◇

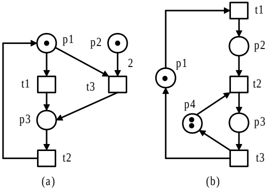


Figure: Two Petri nets, (a) is deadlock-free, (b) is live.

Formal Definitions

Definition

A nonempty set $S \subseteq P$ *is a siphon iff* ${}^\bullet S \subseteq S^\bullet$. $S \subseteq P$ *is a trap iff* $S^\bullet \subseteq {}^\bullet S$. *A siphon (trap) is minimal iff there is no siphon (trap) contained in it as a proper subset. A minimal siphon* S *is said to be strict if* ${}^\bullet S \subsetneq S^\bullet$.

property

Let S_1 *and* S_2 *are two siphons (traps). Then,* $S_1 \cup S_2$ *is a siphon (trap).*

Formal Definitions

Example

$S_1 = \{p_1, p_2, p_3\}$,
$S_2 = \{p_4, p_3\}$,
$S_3 = \{p_2, p_3, p_5\}$, and
$S_4 = \{p_3, p_5\}$ are siphons, among which S_1, S_2, and S_4 are minimal ones. Note that $^\bullet S_1 = S_1^\bullet$, $^\bullet S_2 = S_2^\bullet$, and $^\bullet S_3 = S_3^\bullet$. S_1, S_2, and S_3 are also traps. By $^\bullet S_4 = \{t_2, t_3\}$ and $S_4^\bullet = \{t_1, t_2, t_3\}$, we have $^\bullet S_4 \subset S_4^\bullet$.. ◇

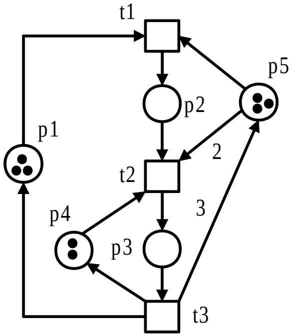


Figure: A Petri net (N, M_0).

Formal Definitions

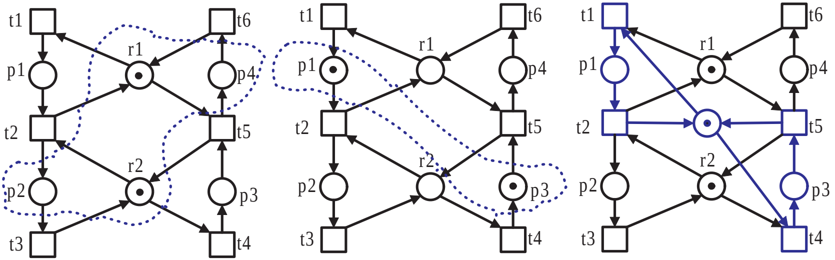


Figure: siphon, token loss, and its control.

Formal Definitions

property

Let $M \in R(N, M_0)$ *be a marking of net* (N, M_0) *and* S *a siphon. If* $M(S) = 0$, *then* $\forall M' \in R(N, M)$, $M'(S) = 0$.

- Once a siphon loses all its tokens, it remains to be unmarked under any subsequent markings that are reachable from the current marking.
- An empty siphon S causes that no transition in $S^\bullet$ is enabled and all transitions connected to S can never be enabled once it is emptied.
- The transitions are therefore dead, leading to the fact that a net containing these transitions is not live.

Formal Definitions

Theorem

Let (N, M_0) be an ordinary net and Π the set of its siphons. The net is deadlock-free if $\forall S \in \Pi$, $\forall M \in R(N, M_0)$, $M(S) > 0$.

This theorem states that an ordinary Petri net is deadlock-free if no (minimal) siphon eventually becomes empty.

Formal Definitions

Theorem

Let (N, M) be an ordinary net that is in a deadlock state. Then, $\{p \in P | M(p) = 0\}$ is a siphon.

This result means that if an ordinary net is dead, i.e., no transition is enabled, then the unmarked places form a siphon.

Formal Definitions

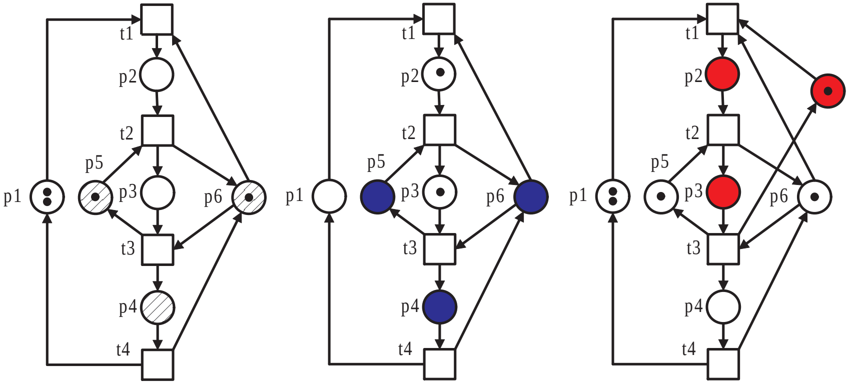


Figure: (a) A Petri net, (b) a dead marking, and (c) a controlled siphon.

Formal Definitions

Example

The net in Fig. **??**(a) has four minimal siphons $S_1 = \{p_1, p_2, p_3, p_4\}$, $S_2 = \{p_3, p_5\}$, $S_3 = \{p_2, p_4, p_6\}$, and $S_4 = \{p_4, p_5, p_6\}$. S_1, S_2 and S_3 are also traps that are initially marked. Note that S_4 is a strict minimal siphon since ${}^\bullet S_4 = \{t_2, t_3, t_4\}$ and $S_4{}^\bullet = \{t_1, t_2, t_3, t_4\}$, leading to the truth of ${}^\bullet S_4 \subset S_4{}^\bullet$.
In Fig. **??**(a), $\sigma = t_1 t_2 t_1$ is a firable transition sequence whose firing leads to a new marking as shown in Fig. **??**(b). The net in Fig. **??**(b) is dead since no transition is enabled in the current marking. The unmarked places p_1, p_4, p_5, and p_6 form a siphon $S = \{p_1, p_4, p_5, p_6\}$ that is not minimal since it contains S_4. The emptiness of S disables every transition in $S^\bullet$ such that no transition in this net is enabled. As a result, the net is dead. ◇

Formal Definitions

Corollary

A deadlocked ordinary Petri net contains at least one empty siphon.

Corollary

Let $N = (P, T, F, W)$ be a deadlocked net under marking M. Then, it has at least one siphon S such that $\forall p \in S$, $\exists t \in p^{\bullet}$ such that $W(p, t) > M(p)$.

Definition

A siphon S is said to be controlled in a net system (N, M_0) iff $\forall M \in R(N, M_0)$, $M(S) > 0$.

Formal Definitions

Definition

Let (N, M_0) be a net system and S be a siphon of N. S is said to be max-marked at a marking $M \in R(N, M_0)$ iff $\exists p \in S$ such that $M(p) \geq max_{p^\bullet}$.

Definition

A siphon is said to be max-controlled iff it is max-marked at any reachable marking.

Definition

(N, M_0) satisfies the cs-property (controlled-siphon property) iff each minimal siphon of N is max-controlled.

Formal Definitions

proposition

Let (N, M_0) be a Petri net and S be a siphon of N. If there exists a P-invariant I such that $\forall p \in (||I||^- \cap S)$, $max_{p^\bullet} = 1$, $||I||^+ \subseteq S$ and $I^T M_0 > \sum_{p \in S} I(p)(max_{p^\bullet} - 1)$, then S is max-controlled.

Formal Definitions

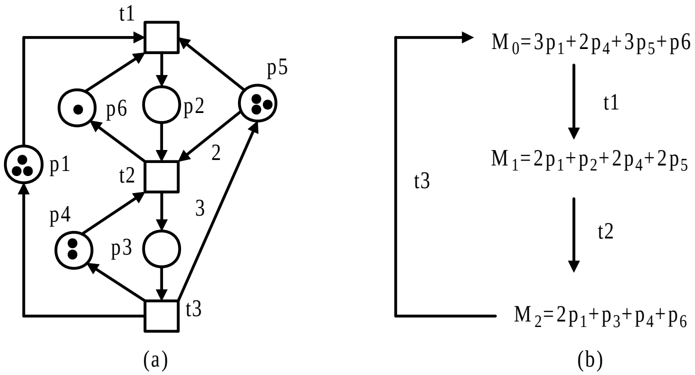


Figure: A max-controlled siphon in a net (N, M_0).

Formal Definitions

Example

Two P-invariants $I_1 = p_2 + p_6$ and $I_2 = p_2 + 3p_3 + p_5$.
$I = I_2 - I_1 = 3p_3 + p_5 - p_6$ is also a P-invariant. $S = \{p_3, p_5\}$ is a strict minimal siphon.
$||I||^- \cap S = \emptyset$ and $||I||^+ = S$.
$I^T M_0 = M_0(p_5) + 3M_0(p_3) - M_0(p_6) = 3 - 1 = 2$.
$\sum_{p \in S} I(p)(max_{p^\bullet} - 1) = I(p_3)(max_{p_3^\bullet} - 1) + I(p_5)(max_{p_5^\bullet} - 1)$.
Considering $max_{p_3^\bullet} = 1$ and $max_{p_5^\bullet} = 2$, we have $\sum_{p \in S} I(p)(max_{p^\bullet} - 1) = 1$. Therefore,
$I^T M_0 > \sum_{p \in S} I(p)(max_{p^\bullet} - 1)$ and S is max-controlled.

Formal Definitions

remark

- *The number of siphons (minimal siphons) grows fast with respect to the size of a Petri net and in the worst case grows exponentially with a net size.*
- *Many deadlock control approaches depend on the complete or partial enumeration of siphons in a plant net model. The complete siphon enumeration is time-consuming. Extensive studies has been conducted on the siphon computation, leading to a variety of methods.*
- *A recent work by Cordone et al. claims that their proposed siphon computation method can find more than 2×10^7 siphons in less than one hour.*

Outline

Elementary and Dependent Siphons

Definition

Let $S \subseteq P$ be a subset of places of Petri net $N = (P, T, F, W)$. P-vector λ_S is called the characteristic P-vector of S iff $\forall p \in S$, $\lambda_S(p)$=1; otherwise $\lambda_S(p) = 0$.

Definition

$\eta_S = [N]^T \lambda_S$ is called the characteristic T-vector of S, where $[N]^T$ is the transpose of incidence matrix $[N]$.

Elementary and Dependent Siphons

Example

Let $S_1 = \{p_1, p_2, p_3, p_4\}$, $S_2 = \{p_2, p_4, p_6\}$, $S_3 = \{p_4, p_5, p_6\}$, and $S_4 = \{p_4, p_6, p_7\}$. We have $\lambda_{S_1} = p_1 + p_2 + p_3 + p_4$, $\lambda_{S_2} = p_2 + p_4 + p_6$, $\lambda_{S_3} = p_4 + p_5 + p_6$, and $\lambda_{S_4} = p_4 + p_6 + p_7$. We have $\eta_{S_1} = \mathbf{0}^T$ and $\eta_{S_2} = \mathbf{0}^T$.

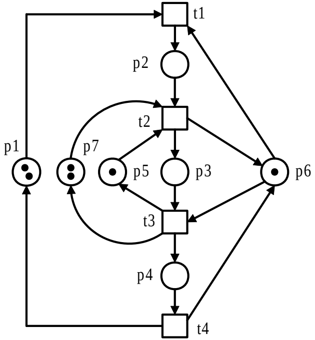


Figure: A Petri net (N, M_0).

Elementary and Dependent Siphons

Example

$S_3 = \{p_4, p_5, p_6\}$,
$\lambda_{S_3} = p_4 + p_5 + p_6$,
$\eta_{S_4} = -t_1 + t_3$.
Suppose a net with 5 transitions.
$\eta_S = -2t_1 + 4t_2 - 3t_4 + t_5$.
Firing t_3 does not change the token count in S.

The physical meanings of a T-vector of a set of place?

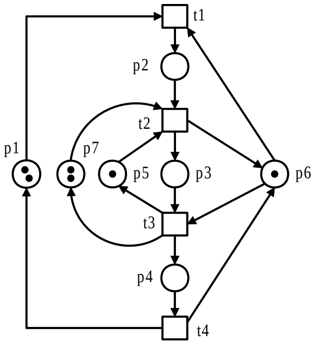


Figure: A Petri net (N, M_0).

Elementary and Dependent Siphons

Definition

Let $N = (P, T, F, W)$ *be a net with* $|P| = m, |T| = n$ *and* $\Pi = \{S_1, S_2, \cdots, S_k\}$ *be a set of siphons of N* $(m, n, k \in \mathbb{N}^+)$. *Let* $\lambda_{S_i}(\eta_{S_i})$ *be the characteristic* $P(T)$*-vector of siphon* $S_i, i \in \mathbb{N}_k$. $[\lambda]_{k\times m} = [\lambda_{S_1}|\lambda_{S_2}|\cdots|\lambda_{S_k}]^T$ *and* $[\eta]_{k\times n} = [\lambda]_{k\times m} \times [N]_{m\times n} = [\eta_{S_1}|\eta_{S_2}|\cdots|\eta_{S_k}]^T$ *are called the characteristic P- and T-vector matrices of the siphons in N, respectively.*

Elementary and Dependent Siphons

Definition

Let η_{S_α}, η_{S_β}, $\cdots$, and η_{S_γ} $(\{\alpha, \beta, \cdots, \gamma\} \subseteq \mathbb{N}_k)$ be a linearly independent maximal set of matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \cdots, S_\gamma\}$ is called a set of elementary siphons in N.

Definition

$S \notin \Pi_E$ is called a strongly dependent siphon if $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$, where $a_i \geq 0$.

Elementary and Dependent Siphons

Definition

$S \notin \Pi_E$ is called a weakly dependent siphon if $\exists A, B \subset \Pi_E$, such that $A \neq \emptyset$, $B \neq \emptyset$, $A \cap B = \emptyset$, and $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_i \in B} a_i \eta_{S_i}$, where $a_i > 0$.

For a weakly dependent siphon S, let $\Gamma^+(S) = \sum_{S_i \in A} a_i \eta_{S_i}$ and $\Gamma^-(S) = \sum_{S_i \in B} a_i \eta_{S_i}$. We have $\eta_S = \Gamma^+(S) - \Gamma^-(S)$. If S is strongly dependent, we define $\Gamma^-(S) = 0$.

Elementary and Dependent Siphons

Definition

Dependent siphons S_1 and S_2 are said to be quasi-equivalent iff $\Gamma^+(S_1) = \Gamma^+(S_2)$.

Lemma

The number of elements in any set of elementary siphons in net N equals the rank of $[\eta]$.

Let Π_E denote a set of the elementary siphons in a Petri net. Since the rank of $[\eta]$ is at most the smaller of $|P|$ and $|T|$, Lemma **??** leads to the following important conclusion.

Elementary and Dependent Siphons

Theorem

$|\Pi_E| \leq min\{|P|, |T|\}$.

This result indicates that the number of elementary siphons in a Petri net is bounded by the smaller of place count and transition count.

Elementary and Dependent Siphons

Example

10 minimal siphons:

$S_1 = \{p_5, p_9, p_{12}, p_{13}\}$,
$S_2 = \{p_4, p_6, p_{13}, p_{14}\}$,
$S_3 = \{p_6, p_9, p_{12}, p_{13}, p_{14}\}$,
$S_4 = \{p_2, p_{15}\}$,
$S_5 = \{p_7, p_{11}\}$,
$S_6 = \{p_1, p_2, p_3, p_5, p_6, p_7\}$,
$S_7 = \{p_4, p_8, p_9, p_{10}\}$,
$S_8 = \{p_3, p_9, p_{12}\}$,
$S_9 = \{p_4, p_5, p_{13}\}$, and
$S_{10} = \{p_6, p_8, p_{14}\}$.

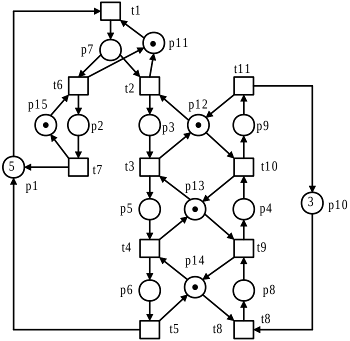


Figure: A Petri net (N_1, M_1).

Elementary and Dependent Siphons

Example

$$\lambda_{S_1} = p_5 + p_9 + p_{12} + p_{13},$$
$$\lambda_{S_2} = p_4 + p_6 + p_{13} + p_{14},$$
$$\lambda_{S_3} = p_6 + p_9 + p_{12} + p_{13} + p_{14},$$
$$\eta_{S_1} = -t_2 + t_3 - t_9 + t_{10},$$
$$\eta_{S_2} = -t_3 + t_4 - t_8 + t_9,$$
$$\eta_{S_3} = -t_2 + t_4 - t_8 + t_{10}.$$

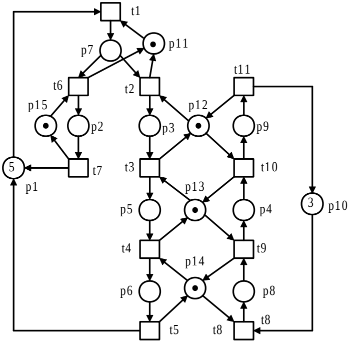


Figure: A Petri net (N_1, M_1).

Elementary and Dependent Siphons

Example

Accordingly, $[\lambda]$ and $[\eta]$ are shown as follows.

$$[\lambda] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$[\eta] = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \end{pmatrix}$$

Elementary and Dependent Siphons

Example

$\eta_{S_3} = \eta_{S_1} + \eta_{S_2}$ and $rank([\eta]) = |\Pi_E| = 2$.

- S_1 and S_2 elementary $\Longrightarrow$ S_3 strongly dependent
- S_1 and S_3 elementary $\Longrightarrow$ S_2 weakly dependent
- S_2 and S_3 elementary $\Longrightarrow$ S_1 weakly dependent

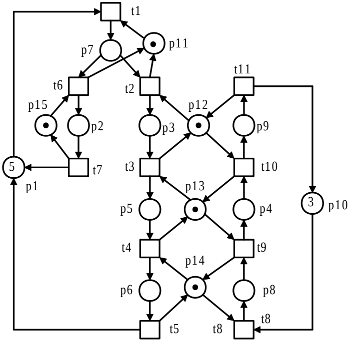


Figure: A Petri net (N_1, M_1).

Outline

Motivation of Classification of Siphons

1. Siphons are classified into elementary and dependent ones
2. The number of elementary ones are bounded by the size of a plant
3. All siphons are controlled by the explicit control of elementary siphons
4. The size of the supervisor is bounded by the size of a plant

Controllability of Dependent Siphons

Let

$$M_{min}(S) = min\{M(S)|M \in R(N, M_0)\} \quad (1)$$

and

$$M_{max}(S) = max\{M(S)|M \in R(N, M_0)\} \quad (2)$$

Controllability of Dependent Siphons

Theorem

Let S be a weakly dependent siphon in net system (N, M_0) with $\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j}$, where $\forall k \in \mathbb{N}_m$, $S_k \in \Pi_E$. S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M_{min}(S_i)) - \sum_{j=n+1}^{m} a_j(M_0(S_j) - M_{max}(S_j)). \quad (3)$$

Controllability of Dependent Siphons

Corollary

Let S be a strongly dependent siphon in net system (N, M_0) with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i},$$

where $\forall i \in \mathbb{N}_n$, $S_i \in \Pi_E$.S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i (M_0(S_i) - M_{min}(S_i)).$$

Controllability of Dependent Siphons

Definition

(N, M_0) *is said to be well-initially-marked iff* $\forall S \in \Pi, M_{max}(S) = M_0(S)$.

It indicates that a siphon of a well-initially-marked Petri net has the maximal number of tokens at the initial marking.

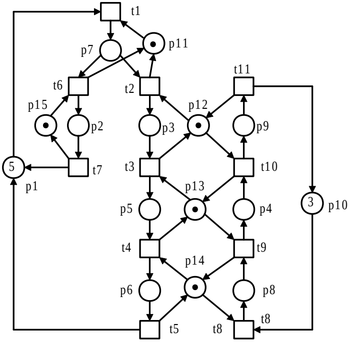


Figure: A Petri net (N_1, M_1).

Controllability of Dependent Siphons

Corollary

Let (N, M_0) be a well-initially-marked net system. A (strongly or weakly) dependent siphon S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M_{min}(S_i)).$$

Controllability of Dependent Siphons

$$M_{min}(S) = min\{M(S)|M \in R(N, M_0)\}$$
$$M_{max}(S) = max\{M(S)|M \in R(N, M_0)\}$$

Let

$$M^{min}(S) = min\{M(S)|M = M_0 + [N]Y, M \geq 0, Y \geq 0\}$$

$$M^{max}(S) = max\{M(S)|M = M_0 + [N]Y, M \geq 0, Y \geq 0\}.$$

$$M^{min}(S) \leq M_{min}(S)$$

$$M^{max}(S) \geq M_{max}(S).$$

Controllability of Dependent Siphons

Corollary

Let S be a weakly dependent siphon in net system (N, M_0) with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j},$$

where $\forall k \in \mathbb{N}_m$, $S_k \in \Pi_E$. *S is controlled if*

$$M_0(S) > \sum_{i=1}^{n} a_i (M_0(S_i) - M^{min}(S_i)) - \sum_{j=n+1}^{m} a_j (M_0(S_j) - M^{max}(S_j)).$$

Controllability of Dependent Siphons

Corollary

Let S be a strongly dependent siphon in net system (N, M_0) with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i},$$

where $\forall k \in \mathbb{N}_n$, $S_k \in \Pi_E$. S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i (M_0(S_i) - M^{min}(S_i)).$$

Controllability of Dependent Siphons

Corollary

Let (N, M_0) be a well-initially-marked net. A (strongly or weakly) dependent siphon S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M^{min}(S_i)).$$

To use them, in the worst case, we need to solve $2|\Pi_E|$ LPP to find $M^{min}(S)$ and $M^{max}(S)$.

Controllability of Dependent Siphons

Define

$$D_1 = min\{\sum_{i=1}^{n} a_i M(S_i) | M = M_0 + [N]Y, M \geq 0, Y \geq 0\}$$

and

$$D_2 = max\{\sum_{j=n+1}^{m} a_j M(S_j) | M = M_0 + [N]Y, M \geq 0, Y \geq 0\}.$$

Next we present weaker conditions under which a dependent siphon can be controlled.

Controllability of Dependent Siphons

Corollary

Let S be a weakly dependent siphon in net system (N, M_0) *with*

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j},$$

where $\forall k \in \mathbb{N}_m$, $S_k \in \Pi_E$*.S is controlled if*

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1 - \sum_{j=n+1}^{m} a_j M_0(S_j) + D_2.$$

Controllability of Dependent Siphons

Corollary

Let S be a strongly dependent siphon in a net system (N, M_0). S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1.$$

Controllability of Dependent Siphons

Corollary

Let S be a (weakly or strongly) dependent siphon in a well-initially-marked net (N, M_0). S is controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1.$$

Controllability of Dependent Siphons

Example

Three strict minimal siphons: S_1={p_5, p_9, p_{12}, p_{13}}, S_2={p_4, p_6, p_{13}, p_{14}}, S_3={p_6, p_9, p_{12}, p_{13}, p_{14}}.
$I_1 = p_5 - p_7 - p_8 + p_9 + p_{12} + p_{13} - V_{S_1}$
$I_2 = -p_3 + p_4 + p_6 - p_7 + p_{13} + p_{14} - V_{S_2}$
are P-invariants.
$S_1 = \{p_5, p_9, p_{12}, p_{13}\}$ is invariant-controlled by I_1 since $\{p|I_1(p) > 0\} = S_1$ and $I_1^T M_2 = M_2(S_1) - M_2(p_7) - M_2(p_8) - M_2(V_{S_1})$=2 − 1 > 0. Likewise, S_2 is controlled by P-invariant I_2.

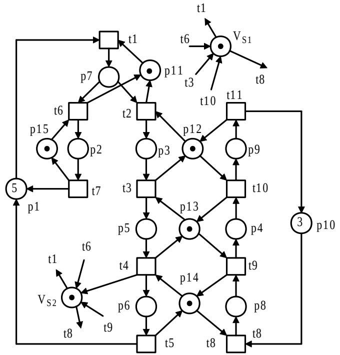


Figure: An augmented net (N_2, M_2).

Controllability of Dependent Siphons

Example

Check the controllability of strongly dependent siphon S_3 with $\eta_{S_3} = \eta_{S_1} + \eta_{S_2}$.
By solving LPP, we have $M^{min}(S_1) = M^{min}(S_2) = 1$. Note that $M_2(S_1) = 2$, $M_2(S_2) = 2$, $M_2(S_3) = 3$, and $\sum_{i=1}^{2}(M_2(S_i) - M^{min}(S_i)) = 2$. $M_2(S_3) > \sum_{i=1}^{2}(M_2(S_i) - M^{min}(S_i))$ S_3 is controlled.

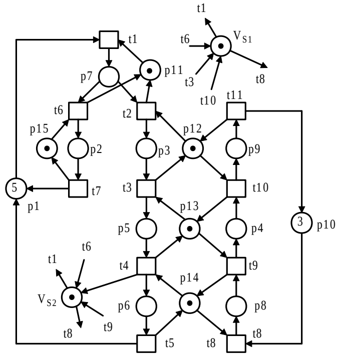


Figure: An augmented net (N_2, M_2).

Controllability of Dependent Siphons

Lemma

Let S be a siphon in net (N, M_0) and $M \in R(N, M_0)$ be a marking. S is max-marked under M if $M(S) > \omega(S)$, where

$$\omega(S) = \sum_{p \in S} (max_{p^\bullet} - 1).$$

$max_{p^\bullet} = max_{t \in p^\bullet}\{W(p, t)\}$

This lemma plays an important role in developing the controllability condition for dependent siphons in a generalized Petri net.

Controllability of Dependent Siphons

Example

The net is a generalized Petri net with three strict minimal siphons

$S_1 = \{p_3, p_6, p_9, p_{13}, p_{14}\}$,
$S_2 = \{p_2, p_5, p_{10}, p_{12}, p_{13}\}$,
$S_3 = \{p_3, p_6, p_{10}, p_{12}, p_{13}, p_{14}\}$.
Thus, we have $\omega(S_1) = 0$, $\omega(S_2) = 1$, and $\omega(S_3) = 1$.

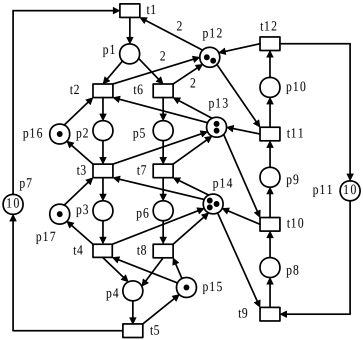


Figure: A generalized Petri net (N, M_0).

Controllability of Dependent Siphons

Theorem

Let (N, M_0), $N = (P, T, F, W)$, *be a marked net system and S be a strongly dependent siphon with*

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i},$$

where $a_i > 0$, $i \in \mathbb{N}_n$.*S is max-controlled if*

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - \sum_{i=1}^{n} a_i M_{min}(S_i) + \omega(S).$$

Controllability of Dependent Siphons

Example

There are three strict minimal siphons. It is easy to verify that S_3 is a strongly dependent siphon with $\eta_{S_3} = \eta_{S_1} + \eta_{S_2}$.

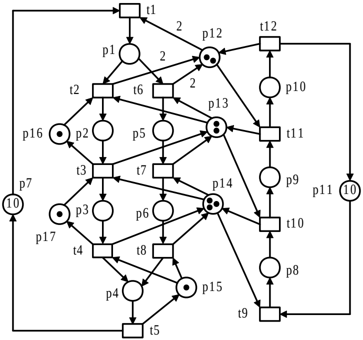


Figure: A generalized Petri net (N, M_0).

Controllability of Dependent Siphons

Example

We have $M_{min}(S_1) = M_{min}(S_2) = 2$ by solving LPP. Considering that $M_1(S_1) = 5$, $M_1(S_2) = 4$, $M_1(S_3) = 7$, and $\omega(S_3) = 1$, $M_1(S_3) > \sum_{i=1}^{2}(M_1(S_i) - M_{min}(S_i)) + \omega(S_3)$ holds. As a result, S_3 is max-controlled. ◇

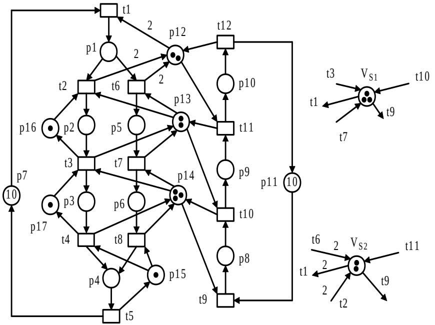


Figure: A siphon is max-controlled in a generalized Petri net (N_1, M_1).

Controllability of Dependent Siphons

Theorem

Let (N, M_0) be a generalized net system and S be a weakly dependent siphon with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j},$$

where $\forall i \in \mathbb{N}_n$, $a_i > 0$ *and* $\forall j \in \{n+1, n+2, \cdots, m\}$, $a_j > 0$.

Controllability of Dependent Siphons

Theorem

S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M_{min}(S_i))$$

$$- \sum_{j=n+1}^{m} a_j(M_0(S_j) - M_{max}(S_j)) + \omega(S).$$

Controllability of Dependent Siphons

Corollary

Let S be a dependent siphon in a well-initially-marked net (N, M_0) with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i}$$

or

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j}.$$

Controllability of Dependent Siphons

Corollary

S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M_{min}(S_i)) + \omega(S).$$

Controllability of Dependent Siphons

Corollary

Let (N, M_0) be a marked net and S be a strongly dependent siphon with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i},$$

where $a_i > 0$, $i \in \mathbb{N}_n$. S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - \sum_{i=1}^{n} a_i M^{min}(S_i) + \omega(S).$$

Controllability of Dependent Siphons

Corollary

Let (N, M_0) be a generalized net and S be a weakly dependent siphon with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j},$$

where $\forall i \in \mathbb{N}_n$, $a_i > 0$ and $\forall j \in \{n+1, n+2, \cdots, m\}$, $a_j > 0$.

Controllability of Dependent Siphons

Corollary

S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S_i) - M^{min}(S_i))$$

$$- \sum_{j=n+1}^{m} a_j(M_0(S_j) - M^{max}(S_j)) + \omega(S).$$

Controllability of Dependent Siphons

Corollary

Let S be a dependent siphon in a well-initially-marked net system. S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i(M_0(S) - M^{min}(S_i)) + \omega(S).$$

Controllability of Dependent Siphons

From the above discussion, in order to verify the controllability of dependent siphons, $\forall i \in \mathbb{N}_{|\Pi_E|}$, we need to compute $M^{min}(S_i)$ and $M^{max}(S_i)$ for S_i. In the worst case, we have to solve $2|\Pi_E|$ LPP.

Controllability of Dependent Siphons

Recalling that

$$D_1 = min\{\sum_{i=1}^{n} a_i M(S_i) | M = M_0 + [N]Y, M \geq 0, Y \geq 0\}$$

and

$$D_2 = max\{\sum_{j=n+1}^{m} a_j M(S_j) | M = M_0 + [N]Y, M \geq 0, Y \geq 0\},$$

we present a better result under which a dependent siphon can be max-controlled.

Controllability of Dependent Siphons

Corollary

Let S be a weakly dependent siphon in net (N, M_0) with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i} - \sum_{j=n+1}^{m} a_j \eta_{S_j},$$

where $\forall k \in \mathbb{N}_m$, $S_k \in \Pi_E$. S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1 - \sum_{j=n+1}^{m} a_j M_0(S_j) + D_2 + \omega(S).$$

Controllability of Dependent Siphons

Corollary

Let S be a strongly dependent siphon with

$$\eta_S = \sum_{i=1}^{n} a_i \eta_{S_i}.$$

S is max-controlled if

$$M_0(S) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1 + \omega(S).$$

Controllability of Dependent Siphons

Corollary

$\omega(S) = 0$ if S is a minimal siphon in an ordinary net.

S^3PR-A Subclass of Petri Nets

Definition

A system of S^2PR, called S^3PR for short, is defined recursively as follows:

- *An S^2PR is an S^3PR.*
- *Let $N_i = (P_{S_i} \cup \{p_i^0\}, T_i, F_i)$, $i \in \{1, 2\}$, be two S^3PR so that $(P_{S_1} \cup \{p_1^0\}) \cap (P_{S_2} \cup \{p_2^0\}) = \emptyset$, $P_{R_1} \cap P_{R_2} = P_C \neq \emptyset$, and $T_1 \cap T_2 = \emptyset$. Then, the net $N = (P_S \cup P^0 \cup P_R, T, F)$ resulting from the composition of N_1 and N_2 via P_C defined as follows: (1) $P_S = P_{S_1} \cup P_{S_2}$, (2) $P^0 = \{p_1^0\} \cup \{p_2^0\}$, (3) $P_R = P_{R_1} \cup P_{R_2}$, (4) $T = T_1 \cup T_2$, and (5) $F = F_1 \cup F_2$ is also an S^3PR.*

S^3PR-A Subclass of Petri Nets

An S^3PR N composed by n S^2PR N_1-N_n, denoted by $N = \bigcirc_{i=1}^{n} N_i$, is defined as follows: $N = N_1$ if $n = 1$; $N = (\bigcirc_{i=1}^{n-1} N_i) \circ N_n$ if $n > 1$. $\overline{N}_i$ is used to denote the S^2P from which the S^2PR N_i is formed. Transitions in $(P^0)^\bullet$ are called source transitions that represent the entry of raw materials when a manufacturing system is modeled with an S^3PR.

S^3PR-A Subclass of Petri Nets

Definition

Let N be an S^3PR. (N, M_0) is called an acceptably marked S^3PR iff one of the following statements is true:

1. (N, M_0) *is an acceptably marked* S^2PR.
2. $N = N_1 \circ N_2$ *so that* (N_i, M_{0_i}) *is an acceptably marked* S^3PR *and*
   (1) $\forall i \in \{1, 2\}, \forall p \in P_{S_i} \cup \{p_i^0\}, M_0(p) = M_{0_i}(p)$.
   (2) $\forall i \in \{1, 2\}, \forall r \in P_{R_i} \setminus P_C, M_0(r) = M_{0_i}(r)$.
   (3) $\forall r \in P_C, M_0(r) = max\{M_{0_1}(r), M_{0_2}(r)\}$.

S^3PR-A Subclass of Petri Nets

property

Let $N = \bigcirc_{i=1}^{n} N_i = (P^0 \cup P_S \cup P_R, T, F)$ *be an* S^3PR *consisting of n simple sequential processes and S be a siphon in N.*
(1) Any $p \in P_{S_i}$ *is associated with a minimal P-semiflow* I_p *with support* $||I_p|| = P_{S_i} \cup \{p_i^0\}$.
(2) Any resource $r \in P_R$ *is associated with a minimal P-semiflow* I_r *such that* $||I_r|| = \{r\} \cup H(r)$.
(3) $\forall p \in [S], \exists r \in S^R, p \in H(r)$ *and* $\forall r' \in P_R \backslash \{r\}, p \notin H(r')$.
(4) $[S] \cup S$ *is the support of a P-semiflow of N.*
(5) $[S] = \cup_{i=1}^{n} [S]^i$, *where* $[S]^i = [S] \cap P_{S_i}$.

S^3PR-A Subclass of Petri Nets

Example

Complementary set
$[S_1] = H(p_{12}) \cup H(p_{13}) \backslash S_1 = \{p_3, p_9, p_5, p_4\} \backslash S_1 = \{p_3, p_4\}$.
Specifically, $[S_1] = [S_1]^1 \cup [S_1]^2$, where $[S_1]^1 = \{p_3\}$ and $[S_1]^2 = \{p_4\}$. The minimal P-invariants associated with idle place p_1 and resource place p_{12} are
$I_{p_1} = p_1 + p_2 + p_3 + p_5 + p_6 + p_7$ and $I_{p_{12}} = p_3 + p_9 + p_{12}$, respectively. $\diamond$

Theorem

An S^3PR (N, M_0) is live iff $\forall M \in R(N, M_0)$, $\forall S \in \Pi$, $M(S) > 0$.

Outline

A Classical Deadlock Prevention Policy

Let $N = (P, T, F)$ be an S^{2}P with idle process place p^0. The length of a path (circuit) in a Petri net is defined as the number of its nodes. The support of a path (circuit) is the set of its nodes.

- Let $\mathcal{C}$ be a circuit of N and x and y be two nodes of $\mathcal{C}$. Node x is said to be *previous* to y iff there exists a path in $\mathcal{C}$ from x to y, the length of which is greater than one and does not pass over the idle process place p^0. This fact is denoted as $x <_{\mathcal{C}} y$.

A Classical Deadlock Prevention Policy

- Let x and y be two nodes in N. Node x is said to be *previous* to y in N iff there exists a circuit $\mathcal{C}$ such that $x <_{\mathcal{C}} y$. This fact is denoted by $x <_N y$.
- Let x and $A \subseteq P \cup T$ be a node and a set of nodes in N, respectively. Then $x <_N A$ iff there exists a node $y \in A$ such that $x <_N y$ and $A <_N x$ iff there exists a node $y \in A$ such that $y <_N x$.

A Classical Deadlock Prevention Policy

Example

In the net N, $\mathcal{C}=p_1 t_1 p_7 t_2 p_3 t_3 p_5 t_4 p_6 t_5 p_1$ is a circuit and $EP(p_7, p_6) = p_7 t_2 p_3 t_3 p_5 t_4 p_6$ is a path in $\mathcal{C}$. The support of $EP(p_7, p_6)$ is $\{p_7, t_2, p_3, t_3, p_5, t_4, p_6\}$ and the support of $\mathcal{C}$ is $\{p_1, t_1, p_7, t_2, p_3, t_3, p_5, t_4, p_6, t_5\}$. Clearly, we have $p_7 <_{\mathcal{C}} p_6$ and $p_7 <_N p_6$. ◇

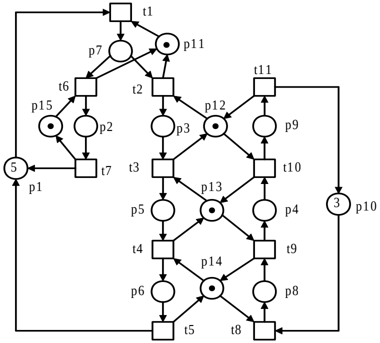


Figure: An S^3PR net (N, M_0).

A Classical Deadlock Prevention Policy

Definition

Let $\Delta^+(t)$ ($\Delta^-(t)$) denote the set of downstream (upstream) siphons of a transition t and $\mathcal{P}_S$ denote the adjoint set of a siphon S in an S^3PR $N = \bigcirc_{i=1}^n N_i = (P^0 \cup P_S \cup P_R, T, F)$.

- *$\Delta^+ : T \rightarrow 2^\Pi$ is a mapping defined as follows: If $t \in T_i$, then $\Delta^+(t) = \{S \in \Pi | t <_{\overline{N}_i} [S]^i\}$. If $S \in \Delta^+(t)$ then the set $[S]^i$ is reachable from t, i.e., there exists a path in $\overline{N}_i$ leading from t to an operation place $p \in P_{S_i}$ that is not included in S but uses a resource of S, where $[S] = \cup_{i=1}^n [S]^i$, $P_S = \cup_{i=1}^n P_{S_i}$, and $[S]^i = [S] \cap P_{S_i}$.*

A Classical Deadlock Prevention Policy

Definition

- $\Delta^{-} : T \rightarrow 2^{\Pi}$ *is a mapping defined as follows: If* $t \in T_i$, *then* $\Delta^{-}(t) = \{S \in \Pi | [S]^i <_{\overline{N}_i} t\}$.
- $\forall i \in \mathbb{N}_n$, $\forall S \in \Pi$, $\mathcal{P}_S^i = [S]^i \cup \{p \in P_{S_i} | p <_{\overline{N}_i} [S]^i\}$, *and* $\mathcal{P}_S = \cup_{i=1}^{n} \mathcal{P}_S^i$.

A Classical Deadlock Prevention Policy

Example

There are three strict minimal siphons $S_1 = \{p_{12}, p_{13}, p_5, p_9\}$, $S_2 = \{p_{13}, p_{14}, p_4, p_6\}$, and $S_3 = \{p_{12}, p_{13}, p_{14}, p_6, p_9\}$. Their complementary sets are $[S_1] = \{p_3, p_4\}$, $[S_2] = \{p_5, p_8\}$, and $[S_3] = \{p_3, p_4, p_5, p_8\}$, respectively. We have downstream siphons $\Delta^+(t_1) = \Delta^+(t_2) = \Delta^+(t_8) = \{S_1, S_2, S_3\}$, $\Delta^+(t_3) = \{S_2, S_3\}$, and $\Delta^+(t_4) = \Delta^+(t_{10}) = \emptyset$.

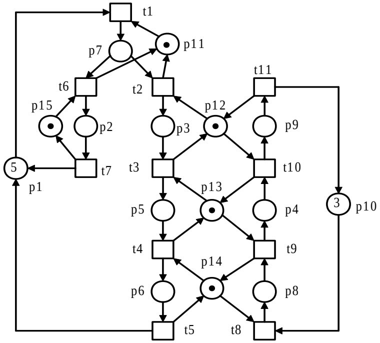


Figure: An S^{3}PR net (N, M_0).

A Classical Deadlock Prevention Policy

Example

Similarly, upstream siphons include $\Delta^-(t_1) = \Delta^-(t_2) = \Delta^-(t_6) = \Delta^-(t_7) = \emptyset$, $\Delta^-(t_3) = \{S_1\}$, and $\Delta^-(t_4) = \Delta^-(t_5) = \{S_1, S_2, S_3\}$. We have adjoint sets $\mathcal{P}_{S_1} = \mathcal{P}^1_{S_1} \cup \mathcal{P}^2_{S_1} = (\{p_3\} \cup \{p_7\}) \cup (\{p_4\} \cup \{p_8\}) = \{p_3, p_4, p_7, p_8\}$, $\mathcal{P}_{S_2} = \mathcal{P}^1_{S_2} \cup \mathcal{P}^2_{S_2} = (\{p_5\} \cup \{p_7, p_3\}) \cup \{p_8\} = \{p_7, p_3, p_5, p_8\}$, and $\mathcal{P}_{S_3} = \mathcal{P}^1_{S_3} \cup \mathcal{P}^2_{S_3} = (\{p_3, p_5\} \cup p_7) \cup \{p_4, p_8\} = \{p_7, p_3, p_5, p_4, p_8\}$.

◇

A Classical Deadlock Prevention Policy

Definition

Let (N, M_0) *be an* S^3PR *with* $N = \bigcirc_{i=1}^{n} N_i = (P_S \cup P^0 \cup P_R, T, F)$. *The net* $(N_A, M_{0A}) = (P_S \cup P^0 \cup P_R \cup P_A, T, F \cup F_A, M_{0A})$ *is the controlled system of* (N, M_0) *iff*

- $P_A = \{V_S | S \in \Pi\}$ *is a set of monitors such that there exists a bijective mapping between* Π *and* P_A.

A Classical Deadlock Prevention Policy

Definition

- $F_A = F_A^1 \cup F_A^2 \cup F_A^3$, *where*
  $F_A^1 = \{(V_S, t) | S \in \Delta^+(t), t \in P^{0\bullet}\}$,
  $F_A^2 = \{(t, V_S) | t \in [S]^\bullet, S \notin \Delta^+(t)\}$, *and*
  $F_A^3 = \cup_{i=1}^n \{(t, V_S) | t \in T_i \setminus P^{0\bullet}, S \notin \Delta^-(t), {}^\bullet t \cap P_{S_i} \subseteq \mathcal{P}_S^i, t \not< [S]^i\}$.
- M_{0A} *is defined as follows: (1)* $\forall p \in P_S \cup P^0 \cup P_R$, $M_{0A}(p) = M_0(p)$ *and (2)* $\forall V_S \in P_A$, $M_{0A}(V_S) = M_0(S) - 1$.

A Classical Deadlock Prevention Policy

Theorem

(N_A, M_{0A}) *is live.*

Example

Three monitors are needed to prevent three strict minimal siphons from being emptied. We first take $S_1 = \{p_{12}, p_{13}, p_5, p_9\}$ as an example. Since $P^0 = \{p_1, p_{10}\}$, we have $P^{0\bullet} = \{t_1, t_8\}$. As a result, $\{(V_{S_1}, t_1), (V_{S_1}, t_8)\} \subseteq F_A^1$.

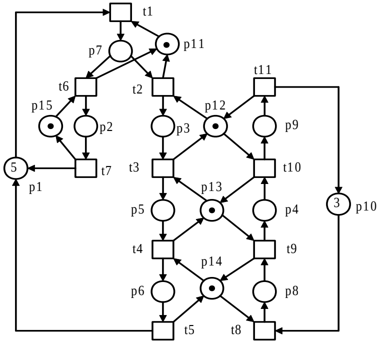


Figure: An S^{3}PR net (N, M_0).

A Classical Deadlock Prevention Policy

Example

Due to $[S_1] = \{p_3, p_4\}$, $[S_1]^\bullet = \{t_3, t_{10}\}$. Note that $S_1 \notin \Delta^+(t_3)$ and $S_1 \notin \Delta^+(t_{10})$. We have $\{(t_3, V_{S_1}), (t_{10}, V_{S_1})\} \subseteq F_A^2$. Next let us find the arcs related to V_{S_1} in F_A^3. Let
$T_\alpha = (T_1 \setminus P^{0^\bullet}) \cup (T_2 \setminus P^{0^\bullet})$, $T_\beta = \{t | S_1 \notin \Delta^-(t), t \in T\}$,
$T_\gamma = \{t | {}^\bullet t \cap P_{S_1} \subseteq \mathcal{P}_S^1\} \cup \{t | {}^\bullet t \cap P_{S_2} \subseteq \mathcal{P}_S^2\}$, and
$T_\delta = \{t | t \nprec [S_1]^1\} \cup \{t | t \nprec [S_1]^2\}$. We have
$T_\alpha = \{t_2, t_3, t_4, t_5, t_6, t_7, t_9, t_{10}, t_{11}\}$, $T_\beta = \{t_1, t_2, t_6, t_7, t_8, t_9\}$,
$T_\gamma = \{t_2, t_3, t_6, t_9, t_{10}\}$, and $T_\delta = \{t_3, t_4, t_5, t_6, t_7, t_{10}, t_{11}\}$. It is easy to see that $T_\alpha \cap T_\beta \cap T_\gamma \cap T_\delta = \{t_6\}$. Consequently, $(t_6, V_{S_1}) \in F_A^3$.

A Classical Deadlock Prevention Policy

Theorem

(N_A, M_{0A}) is live.

Example

For siphons S_2 and S_3, monitors V_{S_2} and V_{S_3} can be added with $\{(V_{S_2}, t_1), (V_{S_2}, t_8), (V_{S_3}, t_1), (V_{S_3}, t_8)\} \subseteq F_A^1$, $\{(t_4, V_{S_2}), (t_9, V_{S_2}), (t_4, V_{S_3}), (t_{10}, V_{S_3})\} \subseteq F_A^2$, and $\{(t_6, V_{S_2}), (t_6, V_{S_3})\} \subseteq F_A^3$. The controlled system for (N, M_0) is shown as right.

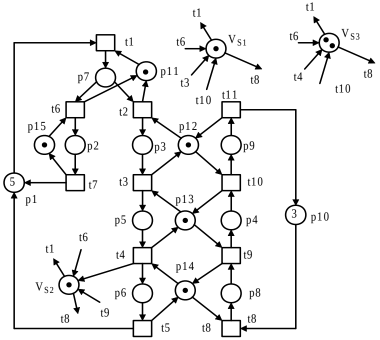


Figure: The controlled system of (N, M_0).

A Classical Deadlock Prevention Policy

remark

For a strict minimal siphon S, this policy ensures that the maximal number of tokens held by $\mathcal{P}_S$ is no more than $M_0(S)$. Since $[S] \subseteq \mathcal{P}_S$, S cannot be emptied if a monitor V_S is added for it. For example, $S_1 = \{p_{12}, p_{13}, p_5, p_9\}$ is a strict minimal siphon with $M_0(S_1) = 2$. Its monitor guarantees the the minimum number of tokens held in $\mathcal{P}_{S_1}$ is $M_0(S_1) - 1 = 1$. ◇

Outline

Elementary Siphon-based Deadlock Prevention Policy

- The number of additional monitors is in theory exponential with respect to the plant net size.
- It involves the complete siphon enumeration.
- The behavior is overly limited.

Elementary Siphon-based Deadlock Prevention Policy

Lemma

Let S and V_S be a siphon and its corresponding monitor, respectively. Then $\forall M \in R(N_A, M_{0A})$, the following invariant relation is verified:

$$M(V_S) + \sum_{i=1}^{n} M(\mathcal{P}_S^i) = M_{0A}(V_S)$$

Elementary Siphon-based Deadlock Prevention Policy

Consider $\cup_{i=1}^{n}\mathcal{P}_S^i = \mathcal{P}_S$ and $\forall i \neq j$, $\mathcal{P}_S^i \cap \mathcal{P}_S^j = \emptyset$, the token invariant relation can be rewritten as $M(V_S) + M(\mathcal{P}_S) = M_{0A}(V_S)$. It is the token invariant relation that ensures the controllability of a strict minimal siphon such that it can never be emptied under any reachable marking in (N_A, M_{0A}).

proposition

Let V_S be a monitor for siphon S in (N_A, M_{0A}). Then $V_S + \sum_{p \in \mathcal{P}_S} p$ is a P-semiflow of N_A.

An Elementary Siphon-based Deadlock Prevention Policy

For a siphon S, a parameter ξ_S, called the control depth variable of the siphon, is introduced in order to establish a flexible siphon control method to facilitate the controllability of a dependent siphon by properly supervising its elementary ones.

proposition

*Let V_S be a monitor computed by Definition **??** for a siphon S in an S^3PR (N, M_0). S is controlled if $M_{0A}(V_S) = M_0(S) - \xi_S$, where $1 \leq \xi_S \leq M_0(S) - 1$.*

An Elementary Siphon-based Deadlock Prevention Policy

Increasing ξ_S intends to tighten the control of siphon S, which may degrade the control performance from the behavior permissiveness point of view. Specifically, a large ξ_S implies that some good (safe) states are possibly removed from the supervisor.

Corollary

Let S be a siphon in an S^3PR and V_S be its monitor defined in Proposition **??**. *In (N_A, M_{0A}), $M_{min}(S) = \xi_S$.*

Elementary Siphon-based Deadlock Prevention Policy

Consider the Petri net with three strict minimal siphons:
$S_1 = \{p_5, p_9, p_{12}, p_{13}\}$,
$S_2 = \{p_4, p_6, p_{13}, p_{14}\}$, and
$S_3 = \{p_6, p_9, p_{12}, p_{13}, p_{14}\}$.

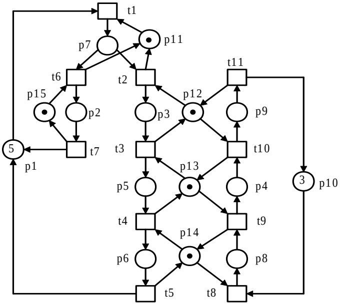


Figure: An S^3PR (N, M_0).

Elementary Siphon-based Deadlock Prevention Policy

The rank of its characteristic T-vector matrix of these siphons equals two, indicating that there are two elementary siphons, the third one is dependent, and $\eta_{S_3} = \eta_{S_1} + \eta_{S_2}$. As a result, we have $\Pi_E = \{S_1, S_2\}$ and $\Pi_D = \{S_3\}$.

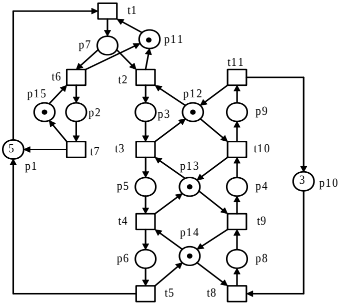


Figure: An S^3PR net (N, M_0).

Elementary Siphon-based Deadlock Prevention Policy

Suppose that V_{S_1} and V_{S_2} are added for S_1 and S_2 with control depth variables ξ_{S_1} and ξ_{S_2}, respectively. Let (N_A, M_{0A}) denote the resultant net with V_{S_1} and V_{S_2}. We then check the controllability of S_3 in (N_A, M_{0A}). According to Corollary **??**, S_3 is controlled if

$$M_{0A}(S_3) > M_{0A}(S_1) + M_{0A}(S_2) - M_{min}(S_1) - M_{min}(S_2)$$

i.e.,

$$M_{0A}(S_3) > M_{0A}(S_1) + M_{0A}(S_2) - \xi_{S_1} - \xi_{S_2}$$

An Elementary Siphon-based Deadlock Prevention Policy

- $M_{0A}(S_1) = M_0(S_1) = 2$, $M_{0A}(S_2) = M_0(S_2) = 2$, and $M_{0A}(S_3) = M_0(S_3) = 3$. The controllability condition is true if $\xi_{S_1} = \xi_{S_2} = 1$.
- This indicates that S_3 is controlled if S_1 and S_2 are controlled by adding monitors V_{S_1} and V_{S_2} with $\xi_{S_1} = \xi_{S_2} = 1$, respectively.
- This also implies that we do not need to add a monitor for S_3 since it has been implicitly controlled due to the controllability of its elementary siphons.

Elementary Siphon-based Deadlock Prevention Policy

Note that the controllability condition is sufficient but not necessary.
Consider a dependent siphon $S_5 = \{p_2, p_4, p_8, p_{10}, p_{17}, p_{21}, p_{22}, p_{26}\}$. It is easy to verify that $\eta_{S_5} = \eta_{S_1} + \eta_{S_3}$, indicating that S_5 is a strongly dependent siphon with respect to S_1 and S_3.

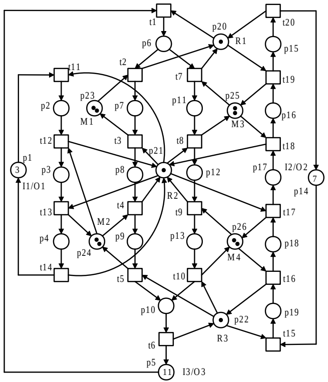


Figure: The Petri net model (N, M_0)

Elementary Siphon-based Deadlock Prevention Policy

Check the controllability of S_5 with $\eta_{S_5} = \eta_{S_1} + \eta_{S_3}$.
Suppose that two monitors V_{S_1} and V_{S_3} are added with $\xi_{S_1} = \xi_{S_3} = 1$.
$M_{0A}(S_5) = M_{0A}(S_1) + M_{0A}(S_3) - \xi_{S_1} - \xi_{S_3}$ if $\xi_{S_1} = \xi_{S_3} = 1$.
Either $\xi_{S_1} = 2$ or $\xi_{S_3} = 2$ will guarantee the controllability of S_5.
How about the controllability of S_5 even if $\xi_{S_1} = \xi_{S_3} = 1$.
Let (N_A, M_{0A}) denote the net that has monitors V_{S_1} and V_{S_3} with $\xi_{S_1} = \xi_{S_3} = 1$. By
$M^{min}(S_5) = min\{M(S_5)|M = M_{0A} + [N_A]Y, M \geq 0, Y \geq 0\}$,
we have $M^{min}(S_5) = 1$. S_5 cannot be emptied in (N_A, M_{0A}) even if $\xi_{S_1} = \xi_{S_3} = 1$. That is to say, we do not need to enlarge any siphon control depth variable in this particular case.

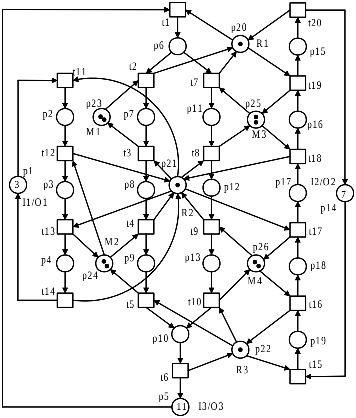
Elementary Siphon-based Deadlock Prevention Policy

This deadlock control policy aims to make a dependent siphon controlled by setting the unit control depth variables of its elementary siphons. When a dependent siphon cannot be controlled by its elementary siphons with their control depth variables being one, a monitor is added for it.

Elementary Siphon-based Deadlock Prevention Policy

Example

There are 18 strict minimal siphons in the net. We have $\Pi_E = \{S_1, S_2, S_3, S_4, S_9, S_{12}\}$ and $\Pi_D = \{S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}\}$.



Elementary Siphon-based Deadlock Prevention Policy

Table: The characteristic T-vector relation between dependent and elementary siphons

S^*	η relationship	S^*	η relationship
S_5	$\eta_{S_5} = \eta_{S_1} + \eta_{S_3}$	S_6	$\eta_{S_6} = \eta_{S_2} + \eta_{S_4}$
S_7	$\eta_{S_7} = \eta_{S_2} + \eta_{S_3}$	S_8	$\eta_{S_8} = \eta_{S_3} + \eta_{S_4}$
S_{10}	$\eta_{S_{10}} = \eta_{S_2} + \eta_{S_3} + \eta_{S_4}$	S_{11}	$\eta_{S_{11}} = \eta_{S_1} + \eta_{S_3} + \eta_{S_4}$
S_{13}	$\eta_{S_{13}} = \eta_{S_4} + \eta_{S_9}$	S_{14}	$\eta_{S_{14}} = \eta_{S_2} + \eta_{S_{12}}$
S_{15}	$\eta_{S_{15}} = \eta_{S_3} + \eta_{S_{12}}$	S_{16}	$\eta_{S_{16}} = \eta_{S_2} + \eta_{S_3} + \eta_{S_{12}}$
S_{17}	$\eta_{S_{17}} = \eta_{S_1} + \eta_{S_3} + \eta_{S_{12}}$	S_{18}	$\eta_{S_{18}} = \eta_{S_9} + \eta_{S_{12}}$

Elementary Siphon-based Deadlock Prevention Policy

Example

- The controllability of siphon S_6 depends on whether $M_0(S_6) > M_0(S_2) + M_0(S_4) - \xi_{S_2} - \xi_{S_4}$ is true. Since $M_0(S_6) = 5$, $M_0(S_2) = 3$, and $M_0(S_4) = 3$, S_6 is controlled if monitors V_{S_2} and V_{S_4} are added when $\xi_{S_2} = \xi_{S_4} = 1$.
- The controllability of S_{16} depends on the truth of $M_0(S_{16}) > M_0(S_2) + M_0(S_3) + M_0(S_{12}) - \xi_{S_2} - \xi_{S_3} - \xi_{S_{12}}$. By $M_0(S_{16}) = 10$, $M_0(S_2) = 3$, $M_0(S_3) = 3$, and $M_0(S_{12}) = 6$, this inequality holds when $\xi_{S_2} = \xi_{S_3} = \xi_{S_{12}} = 1$.

Elementary Siphon-based Deadlock Prevention Policy

Example

It is easy to see that all dependent siphons are controlled by adding six monitors for the elementary siphons only with each siphon control depth variable being unit. That is to say, the addition of six monitors leads to a liveness-enforcing Petri net supervisor for the Petri net model of the FMS. ◇

Elementary Siphon-based Deadlock Prevention Policy

Theorem

Let (N, M_0) be an S^3PR and (N_A, M_{0A}) be the resultant net from adding monitors for m elementary siphons only. (N_A, M_{0A}) is a liveness-enforcing Petri net supervisor with m monitors if the following LPP has a feasible solution:

$$min \sum_{i=1}^{m} \xi_{S_i}$$

Elementary Siphon-based Deadlock Prevention Policy

Theorem

s.t.

$$M_0(S_{Dj}) > \sum_{i=1}^{m} a_i(M_0(S_i) - \xi_{S_i}), j = 1, 2, \cdots, n$$

$$1 \leq \xi_{S_i} \leq M_0(S_i) - 1, i = 1, 2, \cdots, m$$

where $\Pi_D = \{S_{Dj} | j = 1, 2, \cdots, n\}$ *and* $\Pi_E = \{S_i | i = 1, 2, \cdots, m\}$.

Elementary Siphon-based Deadlock Prevention Policy

Example

For the Petri net shown in Fig. **??**, monitors V_{S_1}-V_{S_4}, V_{S_9}, and $V_{S_{12}}$ are added. By solving the following LPP:

$$z = min\{\sum_{i=1}^{4} \xi_{S_i} + \xi_{S_9} + \xi_{S_{12}}\}$$

s.t.

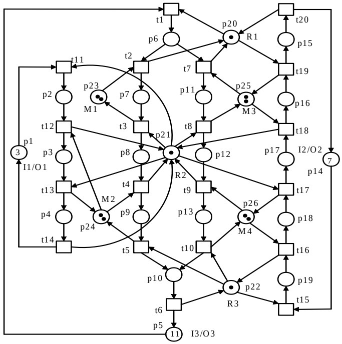


Figure: The Petri net model (N, M_0)

Elementary Siphon-based Deadlock Prevention Policy

Example

$$M_0(S_5) > M_0(S_1) + M_0(S_3) - \xi_{S_1} - \xi_{S_3}$$
$$M_0(S_6) > M_0(S_2) + M_0(S_4) - \xi_{S_2} - \xi_{S_4}$$
$$M_0(S_7) > M_0(S_2) + M_0(S_3) - \xi_{S_2} - \xi_{S_3}$$
$$M_0(S_8) > M_0(S_3) + M_0(S_4) - \xi_{S_3} - \xi_{S_4}$$
$$M_0(S_{10}) > M_0(S_2) + M_0(S_3) + M_0(S_4) - \xi_{S_2} - \xi_{S_3} - \xi_{S_4}$$
$$M_0(S_{11}) > M_0(S_1) + M_0(S_3) + M_0(S_4) - \xi_{S_1} - \xi_{S_3} - \xi_{S_4}$$
$$M_0(S_{13}) > M_0(S_4) + M_0(S_9) - \xi_{S_4} - \xi_{S_9}$$
$$M_0(S_{14}) > M_0(S_2) + M_0(S_{12}) - \xi_{S_2} - \xi_{S_{12}}$$
$$M_0(S_{15}) > M_0(S_3) + M_0(S_{12}) - \xi_{S_3} - \xi_{S_{12}}$$

Elementary Siphon-based Deadlock Prevention Policy

Example

$$M_0(S_{16}) > M_0(S_2) + M_0(S_3) + M_0(S_{12}) - \xi_{S_2} - \xi_{S_3} - \xi_{S_{12}}$$
$$M_0(S_{17}) > M_0(S_1) + M_0(S_3) + M_0(S_{12}) - \xi_{S_1} - \xi_{S_3} - \xi_{S_{12}}$$
$$M_0(S_{18}) > M_0(S_9) + M_0(S_{12}) - \xi_{S_9} - \xi_{S_{12}}$$
$$1 \leq \xi_{S_1} \leq M_0(S_1) - 1$$
$$1 \leq \xi_{S_2} \leq M_0(S_2) - 1$$
$$1 \leq \xi_{S_3} \leq M_0(S_3) - 1$$
$$1 \leq \xi_{S_4} \leq M_0(S_4) - 1$$
$$1 \leq \xi_{S_9} \leq M_0(S_9) - 1$$
$$1 \leq \xi_{S_{12}} \leq M_0(S_{12}) - 1$$

Two Liveness-enforcing Supervisors

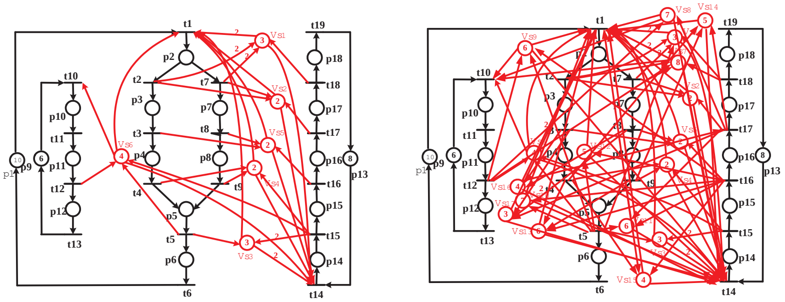


Figure: Two supervisors for (N, M_0)

Supervisor structure for different-sized systems

Examples	number of resources	No. part -types	plant net model size	additions due to [8]	additions by the proposed idea
Example 1	3 machines 2 robots	2	$[P,T,F] = [15,11,39]$	3 monitors 15 arcs	2 monitors 10 arcs
Example 2	4 machines 3 robots	3	$[P,T,F] = [26,20,74]$	18 monitors 106 arcs	6 monitors 32 arcs
Example 3	7 machines 5 robots	5	$[P,T,F] = [48,38,142]$	70 monitors 686 arcs	10 monitors 66 arcs
Example 4	10 machines 7 robots	7	$[P,T,F] = [68,54,203]$	169 monitors 2255 arcs	13 monitors 81 arcs
Example 5	13 machines 9 robots	10	$[P,T,F] = [90,68,285]$	327 monitors 6439 arcs	21 monitors 116 arcs
Example 6	16 machines 13 robots	13	$[P,T,F] = [128,88,372]$	587 monitors 15464 arcs	32 monitors 186 arcs

Figure: Supervisor structure for different-sized systems

Supervisor structure for different-sized systems

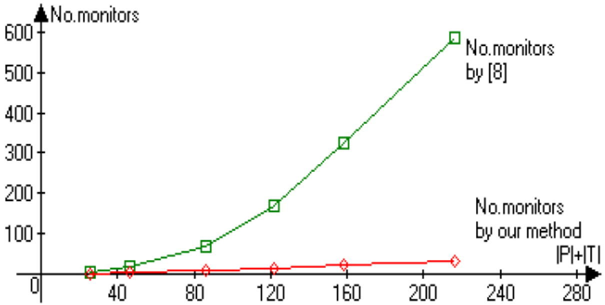


Figure: Supervisor structure for different-sized systems

Supervisor structure for different-sized systems

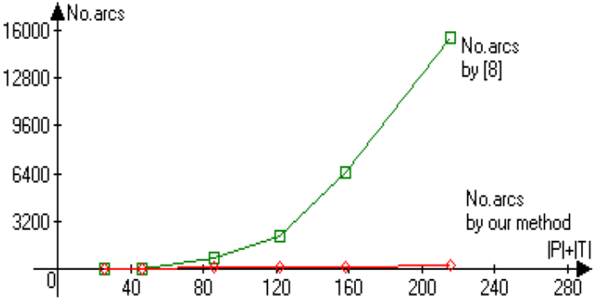


Figure: Supervisor structure for different-sized systems

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

Insufficient marked resource places in a Petri net can lead to deadlock states. They can be prevented by the proper control of a special structural object called siphons. An empty siphon is closely tied to the existence of dead transitions in an ordinary Petri net while in a generalized Petri net, an insufficiently marked siphon can lead to a dead state. Many deadlock control policies for generalized Petri nets are developed based on the concept of max-controlled siphons. This control condition is only sufficient but not necessary.

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

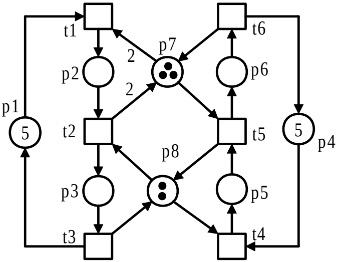


Figure: A live net

Example

- The net is live.
- $S = \{p_3, p_6, p_7, p_8\}$ is a strict minimal siphon. Since $max_{p_7^\bullet} = 2$, $max_{p_8^\bullet} = 1$, at marking $M = 4p_1 + p_2 + 3p_4 + 2p_5 + p_7$, siphon S not max-marked.

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

Definition

Let S be a siphon of a well-marked S^{4}R net (N, M_0). S is said to be max′-marked at marking $M \in R(N, M_0)$ if $\exists p \in S^P$ such that $M(p) \geq 1$ or $\exists r \in S^R$ such that $M(r) \geq max_{t\in(r^\bullet \cap [S]^\bullet)}\{W(r, t)\}$.

Definition

Let S be a siphon of a well-marked S^{4}R net (N, M_0). S is said to be max′-controlled if S is max′-marked at any reachable marking, that is, $\forall M \in R(N, M_0)$, $\exists p \in S^P$ such that $M(p) \geq 1$ or $\exists r \in S^R$ such that $M(r) \geq max_{t\in(r^\bullet \cap [S]^\bullet)}\{W(r, t)\}$.

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

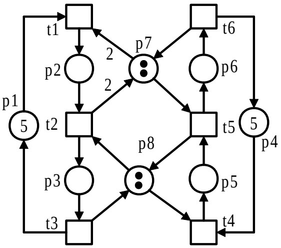
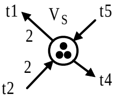


Figure: A live net



Example

- Siphon $S = \{p_3, p_6, p_7, p_8\}$ is a strict minimal siphon. $[S] = \{p_2, p_5\}$, $[S]^\bullet = \{t_2, t_5\}$, $p_7^\bullet = \{t_1, t_5\}$, $[S]^\bullet \cap p_7^\bullet = \{t_5\}$, $p_8^\bullet = \{t_2, t_4\}$, $[S]^\bullet \cap p_8^\bullet = \{t_2\}$, $max_{t \in (p_7^\bullet \cap [S]^\bullet)}\{W(p_7, t)\} = W(p_7, t_5) = 1$, $max_{t \in (p_8^\bullet \cap [S]^\bullet)}\{W(p_8, t)\} = W(p_8, t_2) = 1$.
- S is max′-controlled when monitor V_S is added to the net.

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

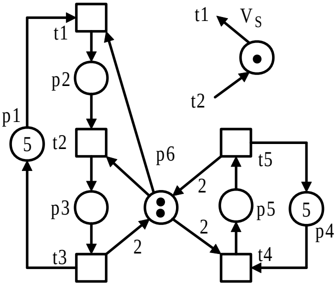


Figure: A live net

Example

- Siphon $S = \{p_3, p_5, p_6\}$ is a strict minimal siphon. $[S] = \{p_2\}$, $[S]^\bullet = \{t_2\}$, $p_6^\bullet = \{t_1, t_2, t_4\}$, $[S]^\bullet \cap p_6^\bullet = \{t_2\}$, $max_{t \in (p_6^\bullet \cap [S]^\bullet)}\{W(p_6, t)\} = W(p_6, t_2) = 1$.
- S is max′-controlled when monitor V_S is added to the net.
- Since $max_{p_6^\bullet} = 2$, given a marking M, if $M(p_2) = 1$, S is not max-marked at M. Then t_1 is forbidden to fire if S is max-controlled.

Siphon Control in Generalized Petri Nets: Max′-controlled Siphon

Since $max_{r^\bullet} \geq max_{t \in (r^\bullet \cap [S]^\bullet)}\{W(r,t)\}$, a max-controlled siphon is a max′-controlled siphon. However, a max′-controlled siphon may not be a max-controlled siphon.

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

Max$'$-controlled condition of siphon is still a sufficient one but not necessary

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

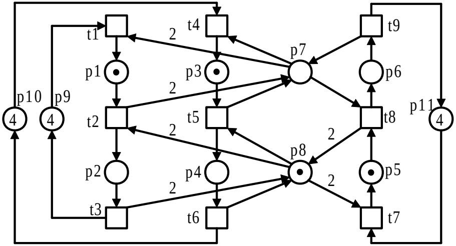


Figure: A live S^{4}R net with a non-max$'$-controlled siphon.

Example

- $S = \{p_2, p_4, p_6, p_7, p_8\}$ is a strict minimal siphon.
  $[S]^\bullet = \{t_2, t_5, t_8\}$,
  $p_7^\bullet = \{t_1, t_4, t_8\}$, $p_7^\bullet \cap [S]^\bullet = \{t_8\}$,
  $p_8^\bullet = \{t_2, t_5, t_7\}$,
  $p_8^\bullet \cap [S]^\bullet = \{t_2, t_5\}$,
  $W(p_7, t_8) = 1$, $W(p_8, t_2) = 2$.
- S is not max$'$-marked at $M = p_1 + p_3 + p_5 + p_8 + 4p_9 + 4p_{10} + 4p_{11}$.

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

Definition

Let S be a siphon in a well-marked S^{4}R (N, M_0). S is said to be max$''$-marked at $M \in R(N, M_0)$ if at least one of the following conditions holds:

- (i) M is an initial marking;
- (ii) $\exists p \in S^P$ such that $M(p) \geq 1$;
- (iii) $\exists r \in S^R$ such that $\exists t \in T'$, $T' = \{t | t \in r^\bullet \cap [S]^\bullet,\ M(r) \geq W(r,t),\ M(P_S \cap {}^\bullet t) \geq 1\}$, and if $\exists r' \in S^R \cap t^\bullet$, then $\sum_{t \in T'} M(P_S \cap {}^\bullet t) \cdot W(t, r') + M(r') \geq max_{t' \in r'^\bullet \cap [S]^\bullet} \{W(r', t')\}$.

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

From the Definition of max$''$-marked siphon, given a marking M, a max$''$-marked siphon can guarantee that at least one transition in its postset can fire once.

Definition

Let S be a siphon in a well-marked S^{4}R (N, M_0). S is said to be max$''$-controlled if $\forall M \in R(N, M_0)$, S is max$''$-marked at M.

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

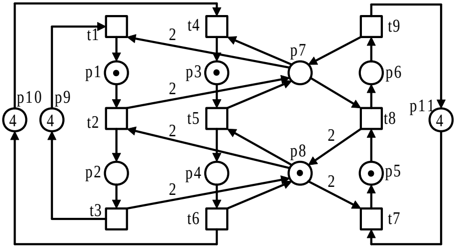


Figure: A live S[4]R net with a max$''$-controlled siphon.

Example

- $S = \{p_2, p_4, p_6, p_7, p_8\}$, $S^P = \{p_2, p_4, p_6\}$, $S^R = \{p_7, p_8\}$.
- $M = p_1 + p_3 + p_5 + p_8 + 4p_9 + 4p_{10} + 4p_{11}$.
- Note that $t_5 \in p_8^\bullet \cap [S]^\bullet$ and $p_3 \in {}^\bullet t_5 \cap P_A$. We have $M(p_8) = M(p_3) = 1$ and $M(p_3) \cdot W(t_5, p_7) + M(p_7) = 1 \times 1 + 0 = 1 = W(p_7, t_8)$. Hence, S is max$''$-marked at M.

Siphon Control in Generalized Petri Nets: Max″-controlled Siphon

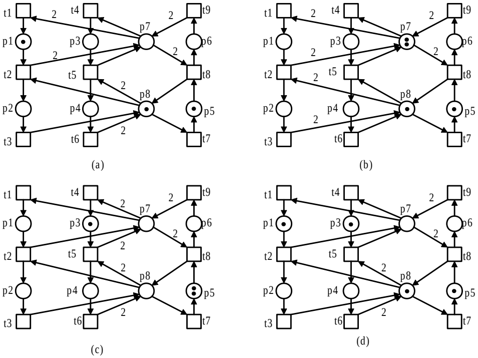


Figure: (a) and (b), S is max″-marked, and (c) and (d), S is non-max″-marked at the shown markings.

Siphon Control in Generalized Petri Nets: Max″-controlled Siphon

As we know, some siphons in an S^{4}R may contain only one resource place but one or more operation places. In such a case, r' and t' do not exist. Thus we do not need to check the condition $\sum_{t\in T'} M(P_A \cap {}^\bullet t) \cdot W(t, r') + M(r') \geq max_{t' \in r'^\bullet \cap [S]^\bullet}\{W(r', t')\}$ any more.

Siphon Control in Generalized Petri Nets: Max″-controlled Siphon

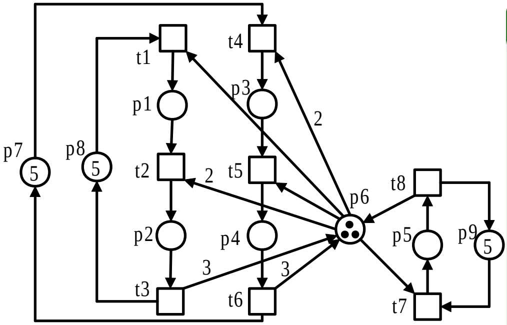


Figure: Example for a siphon with only a resource place.

Example

A well-marked S^4R net model (N, M_0) has a unique SMS $S = \{p_2, p_4, p_5, p_6\}$. Together with $H(p_6) = \{p_1, p_2, p_3, p_4, p_5\}$, we can find $[S] = \{p_1, p_3\}$ and $p_6^\bullet \cap [S]^\bullet = \{t_2, t_5\}$. By firing t_4 once at the initial marking, we have a new M with $M(p_3) = M(p_6) = 1$ at which S is still max″-marked.

Siphon Control in Generalized Petri Nets: Max$''$-controlled Siphon

A sufficient and necessary control condition for siphons in generalized Petri net is still an open problem.

Conjecture

- The existence of polynomial deadlock prevention algorithm based on elementary siphons
- polynomial algorithm of computing elementary siphons without the complete siphon enumeration
- the proof that all siphons can be controlled by supervising elementary siphons only

Behavioral permissiveness, structural complexity, and computational complexity are the most important criteria to evaluate the performance of a supervisor.

- Major Technical Obstacles
  - Computationally efficient methods of siphons and complete state enumeration
- Structural Analysis
  - Find elegant supervisor (behaviorally optimal and structurally minimal) by structural analysi

Thanks!

Comments, Suggestions, and Questions?