Minimization for ATL* models

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This work: what and why

- What: proposition of an algorithm minimizing the number of states of a model for an ATL* formula A. Minimizing w.r.t. alternating bisimulation;
- Why: big models are difficult to handle (to model-check properties other than A, for instance);
- Intended applications: among others, improvement of TATL, the online software that constructively decides the satisfiability ATL* formulae by tableaux (A. David).

ATL*: Alternating-time Temporal Logic

- Extends CTL* to the multi-agent case.
- Several agents act synchronously, and can form coalitions.
- CTL's existential quantification E - there is a path - becomes $\langle\langle A \rangle\rangle$, A being a coalition of agents: it ranges over collective strategies of A.
- Intuitively $\langle\langle A \rangle\rangle \Phi$ means : there is a strategy for the coalition A that assures Φ, no matter how the other agents play.

ATL*: Alternating-time Temporal Logic

- ATL* formulae

  State formulae :
  $\psi := p \mid True \mid (\neg\psi) \mid (\psi \wedge \psi) \mid (\langle\langle A \rangle\rangle \Phi)$
  where p is a proposition.

  Path formulae :
  $\Phi := \psi \mid (\neg\Phi) \mid (\Phi \wedge \Phi) \mid (\bigcirc\Phi) \mid (\Box\Phi) \mid (\Phi U \Phi)$

$$\Diamond\Phi \;=_{def}\; True U \Phi$$

- $\bigcirc$: at the next state
  $\Box$: now and in the future
  U: until
  $\Diamond$: now or in the future

Concurrent Game Model (CGM)

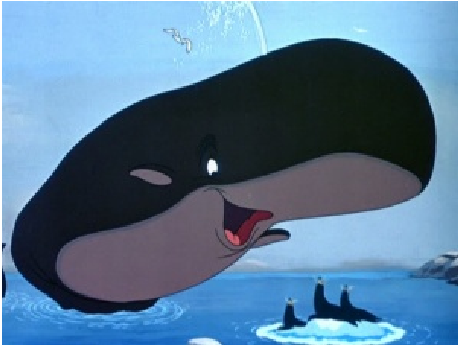
Formulae are interpreted over Concurrent Game Models (CGMs), that are labeled graphs (labels at states and edges).

Six components of a CGM:

1. A finite non-empty set of *agents*: $\mathbb{A} = \{1, ..., k\}$;
2. A non-empty set of states $\mathbb{S}$;
3. A non empty set of actions;
4. Functions act_i mapping each state to the set of actions available at that state, for each given agent i;
5. A transition function out connecting each state to successor states via global moves $=$ vectors of synchronous actions of **all** the agents;
6. A labelling function L associating to each state s the set of propositions holding at s.

A CGM example through a fairy tale

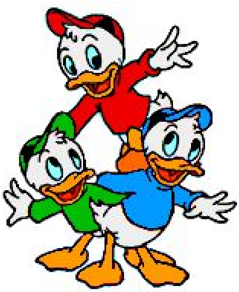
Once upon a time there was a gentle whale, who lived happily in deep sea.



But a bad day she got stuck on a shore. She was in great danger.

A CGM example through a fairy tale

Three little ducks decided to help her, by pushing her away from the shore, towards deep water. Their names: GP1, GP2 and GP3.



But each one of them, alone, was not very strong, and...

A CGM example through a fairy tale

And a very nasty, big and strong duck, named Nasty Donald, wanted the extinctions of whales. So, he was fighting GP1, GP2 and GP3, pushing the whale back to the shore.

A CGM example through a fairy tale

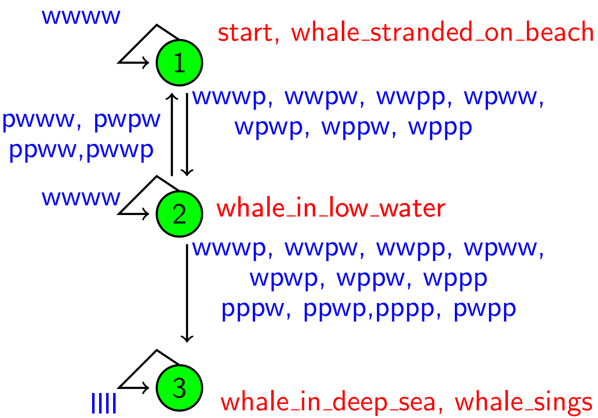
$GP1$, $GP2$ and $GP3$ decide to form a coalition. If they adopt the strategy to push together, at least two of them at the same time, they will be strong enough to overcome Nasty Donald.

The whale will get to deep water, she will be save, and she will happily sing.

Nasty Donald CGM: agents, states, allowed actions

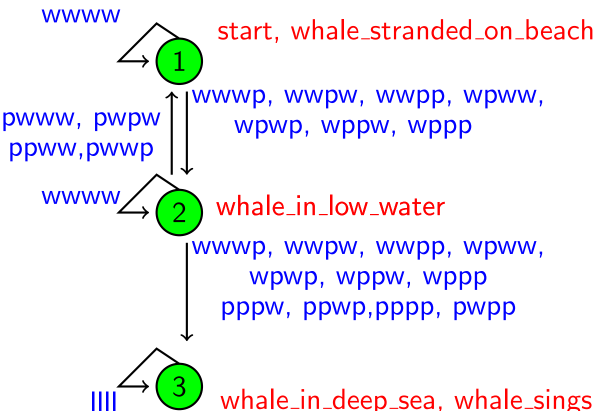
- Four agents : Nasty Donald (ND), GP1, GP2 and GP3. Say: agents 1, 2, 3 and 4 (in this order).
- Three states: 1, 2 and 3.
- Only true propositional variables at 1: start (initial situation, to which it is possible to can come back) and whale_stranded_on_beach. Only true propositional variable at 2: whale_in_low_water. Only true propositional variables at 3 : whale_in_deep_see, whale_sings.
- At state 1 Nasty Donald can only wait, while GP1, GP2 and GP3 can do the actions wait or push the whale (towards the deep sea). At state 2 each agent can wait or push the whale (GP1, GP2 and GP3 towards the deep sea, Nasty Donald towards the shore). At state 3, everybody can just look at what happens.

Nasty Donald CGM: transitions



Nasty Donald CGM: true formulae at state 1

$\langle\langle GP1, GP2 \rangle\rangle \Diamond \mathit{whale\_in\_deep\_sea}$,
$\neg \langle\langle GP1 \rangle\rangle \Diamond \mathit{whale\_in\_deep\_sea}$,
$\neg \langle\langle ND \rangle\rangle \Box \mathit{whale\_stranded\_on\_beach}$,
$\langle\langle GP1, GP2 \rangle\rangle \Diamond \Box \mathit{whale\_in\_deep\_sea}$,



Our approach to minimization of a CGM

- To reduce the number of states of a CGM $\mathcal{M}_1$ satisfying an ATL* formula F, we build a CGM $\mathcal{M}_2$ that is bisimilar to $\mathcal{M}_1$.
- $\mathcal{M}_1$ bisimilar to $\mathcal{M}_2$: intuitively, actions in $\mathcal{M}_1$ are simulated by actions in $\mathcal{M}_2$, and viceversa. The models $\mathcal{M}_1$ and $\mathcal{M}_2$ satisfy the same formulae at bisimilar states, thus $\mathcal{M}_2$ too is a model of F.
- Moreover, we build $\mathcal{M}_2$ so that it is minimal (w.r.t. number of states)
  among the CGMs that are bisimilar to $\mathcal{M}_1$.

Alternating A-bisimulation between models $\mathcal{M}_1$ and $\mathcal{M}_2$

CGM's $\mathcal{M}_1$ and $\mathcal{M}_2$: same propositions, same agents.
States of $\mathcal{M}_1$: $\mathbb{S}_1$, states of $\mathcal{M}_2$: $\mathbb{S}_2$.
A: a coalition of agents (i.e. : a set of agents, acting synchronously).

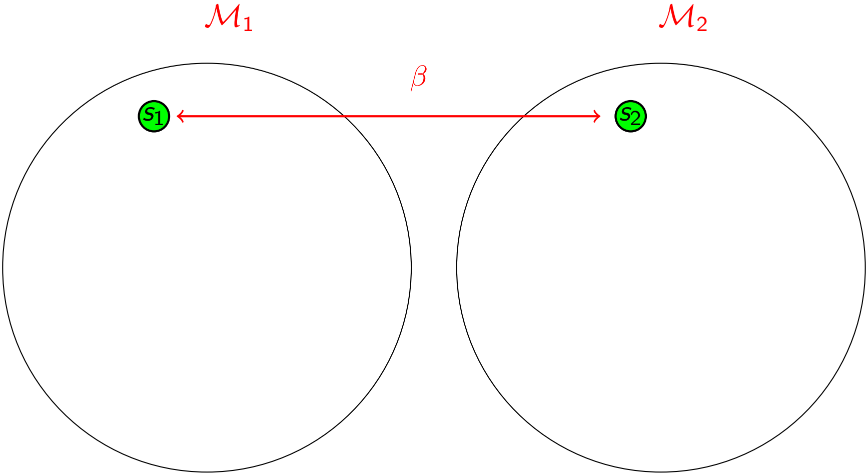
DEF[Ågotnes, Goranko and Jamroga 2007, modifying Alur *et al.* 1998]

A relation $\beta \subseteq \mathbb{S}_1 \times \mathbb{S}_2$ is an *alternating A-bisimulation* iff $\forall s_1 \in states_1$, $\forall s_2 \in states_2$, **IF** $s_1 \beta s_2$ **THEN**:

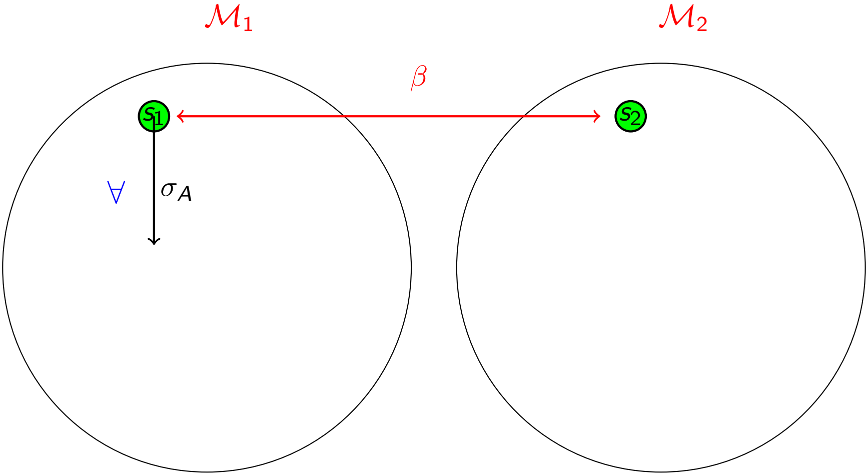
1. Exactly the same boolean variables are true at s_1 and s_2;
2. The *Forth Condition* and the *Back Condition* hold.

What do these conditions say ?

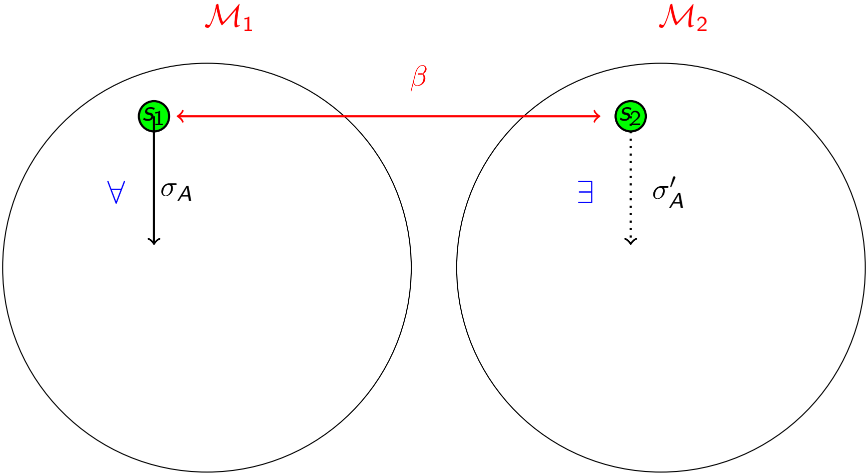
Forth Condition in a picture



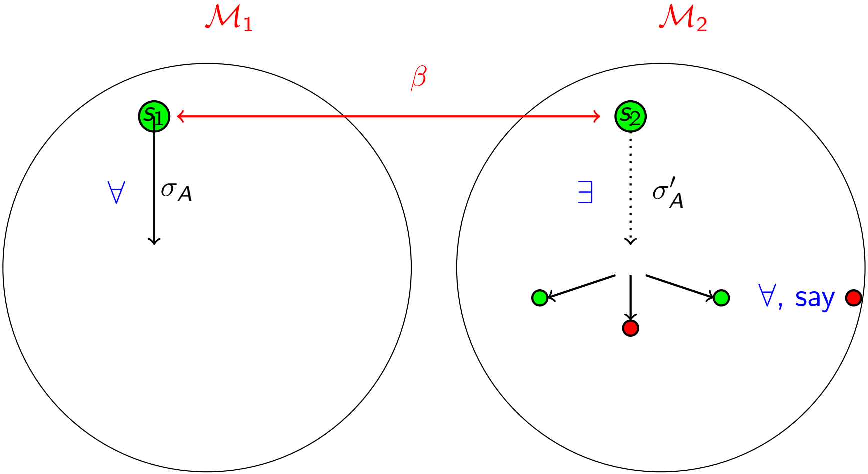
Forth Condition in a picture



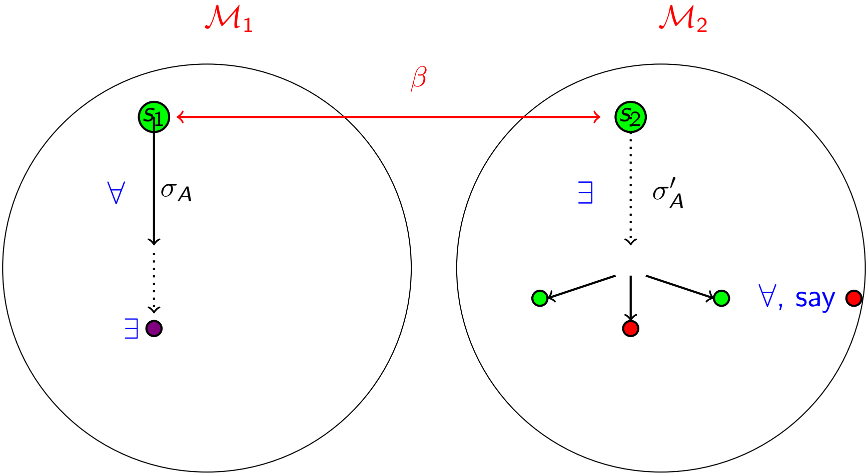
Forth Condition in a picture



Forth Condition in a picture

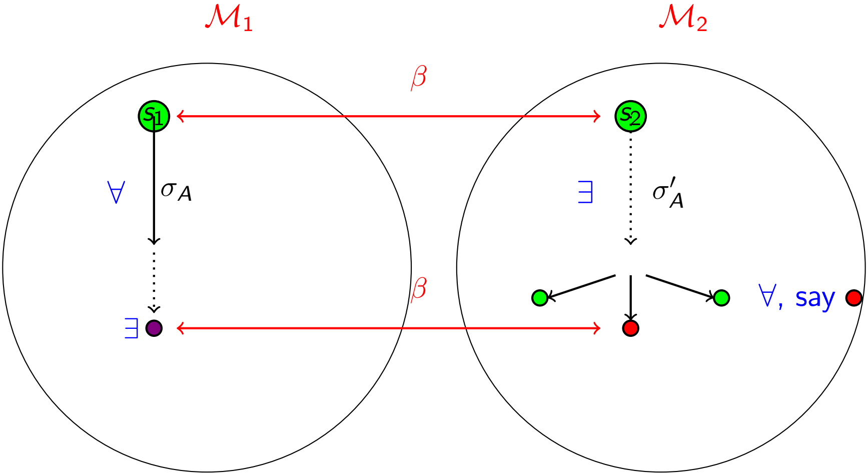


Forth Condition in a picture



such that:

Forth Condition in a picture



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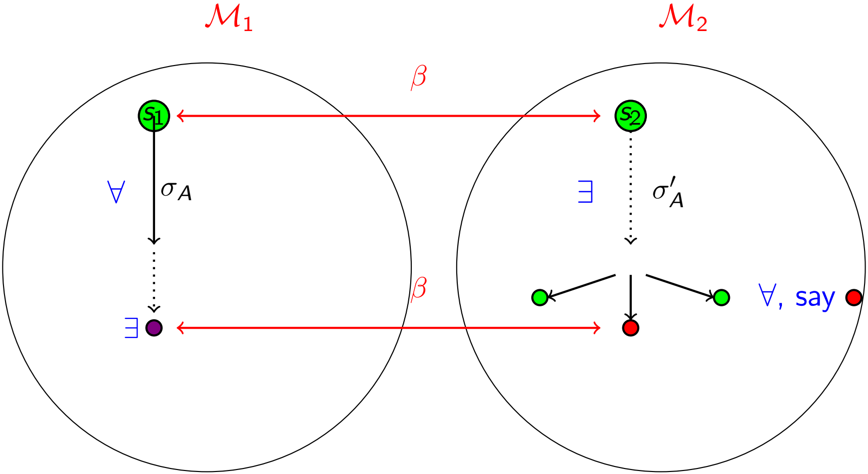


The A-move σ'_A simulates σ_A

Back Condition, Bisimilar Models

As the Forth one, but the other way around, from $\mathcal{M}_2$ to $\mathcal{M}_1$.
Bisimulation.

If there is a $\beta \subseteq \mathbb{S}_1 \times \mathbb{S}_2$ that is an A-alternating bisimulation for each coalition A, then $\mathcal{M}_1$ and $\mathcal{M}_2$ are (fully) alternating bisimilar.

This definition is co-inductive.
It isn't constructive and seems circular.
But...

Toward **computing** a model bisimilar to $\mathcal{M}_1$

The construction of a model (fully) alternating bisimilar to an input $\mathcal{M}_1$ can be done stepwise, by successive approximations (as for labelled transition systems).

Actually we construct a model that is minimal (in the number of states) w.r.t. (full) bisimulation with $\mathcal{M}_1$.

We use a "local notion": Behavioural equivalence of states w.r.t a partition of the set of states of the input model $\mathcal{M}_1$. Provided that P is *harmonious*: if s_1 and s_2 are in the same cluster of P then they verify the same propositions.

Behavioural equivalence of states wrt a partition: $\equiv_{PA}$

P: a harmonious partition of the states of $\mathcal{M}_1$,
s and t in the same cluster, A a coalition of agents.

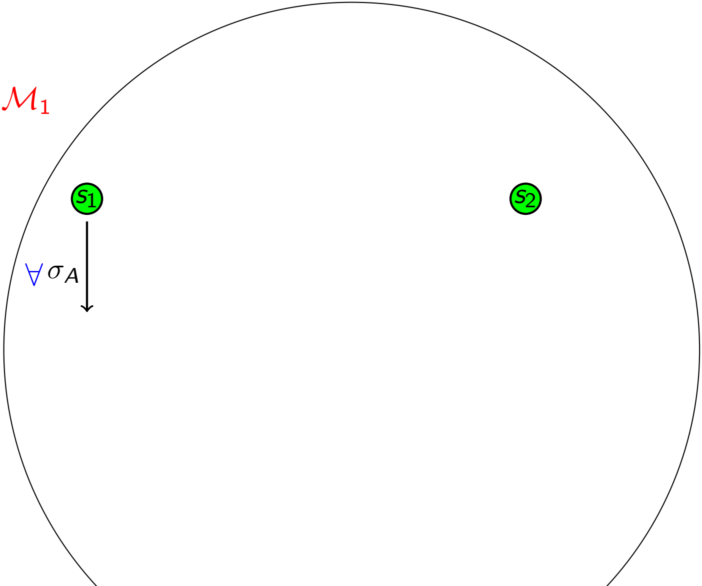
Our definition:

The states s_1 and s_2 are *behaviourally A-equivalent w.r.t. P* ($s_1 \equiv_{P^A} s_2$) when :

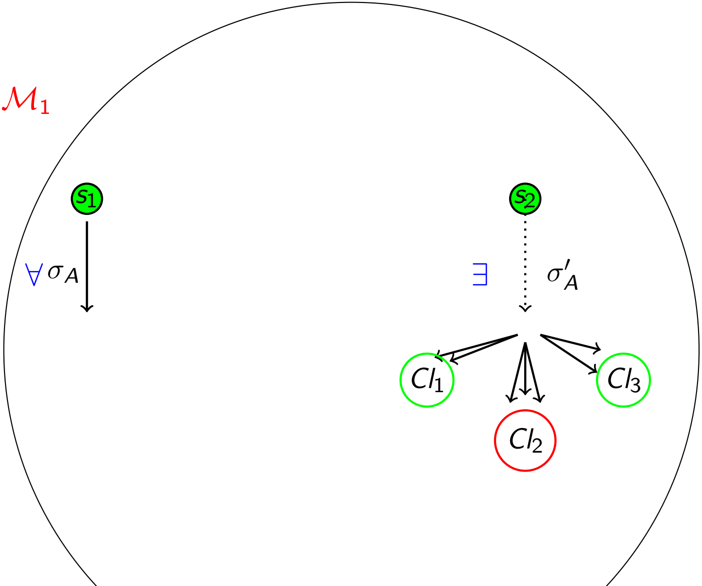
- Given any action σ_A available at s_1, there is an action σ'_A available at s_2 such that the set of clusters of states that are reachable from s_2 via σ'_A is a subset of the set of clusters of states that are reachable from s_1 via σ_A.
- The other way around.

The states s_1 and s_2 are *behaviourally equivalent w.r.t. P* iff $s_1 \equiv_{P^A} s_2$ for **each** coalition A.

$s_1 \equiv_{PA} s_2$ in a picture



$s_1 \equiv_{PA} s_2$ in a picture



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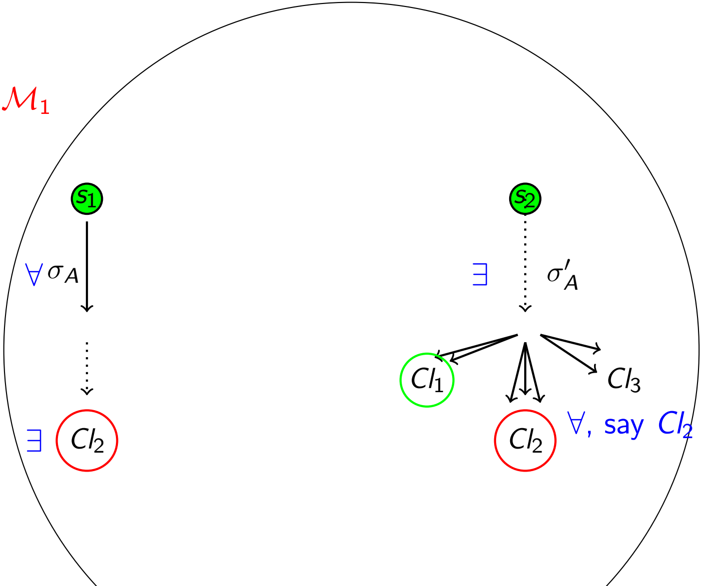

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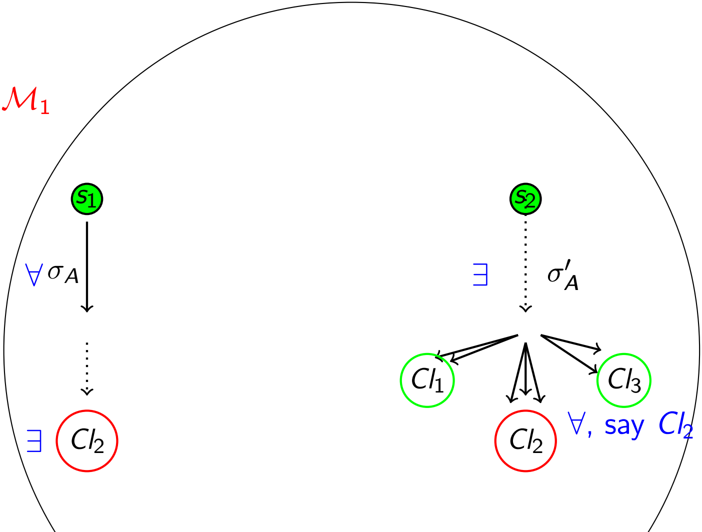


and the other way around, from s_2 to s_1

Minimization algorithm: Specification

- Input: a CGM $\mathcal{M}_1$.
- Output: a CGM $\mathcal{M}_2$ that is:
  1. Fully bisimilar to $\mathcal{M}_1$;
  2. Smaller (in the number of states) than $\mathcal{M}_1$, actually minimal w.r.t. bisimulation with $\mathcal{M}_1$.

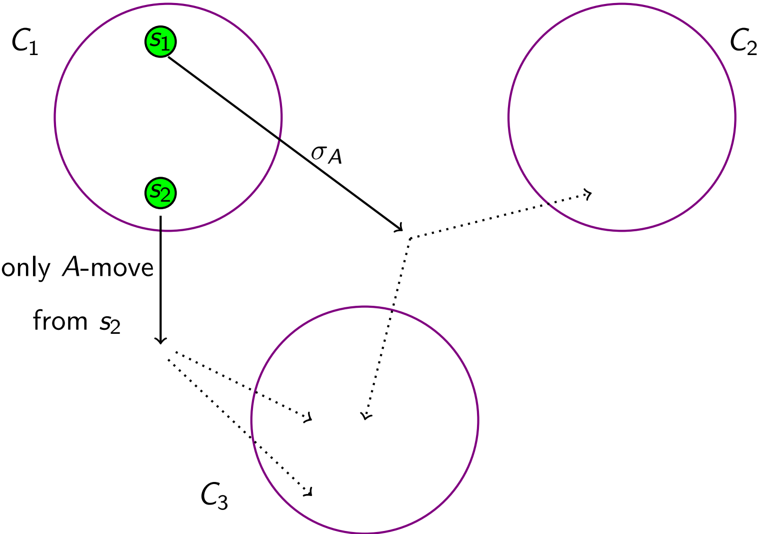
Minimization algorithm: Principles

$\mathbb{S}_1$ = the states of the input model $\mathcal{M}_1$.

1. Partition $\mathbb{S}_1$ putting two states in the same cluster iff they make true the same propositions: initial *harmonious* partition;
2. Refine the current partition by splitting a cluster C whenever there are a coalition A and states s and t in C such that $s \not\equiv_{P^A} t$.
   Do this until stability (no more splits possible).
3. If $\beta \subseteq \mathbb{S}_1 \times \mathbb{S}_1$ is the (equivalence) relation corresponding to the final partition of $\mathbb{S}_1$, build the quotient of $\mathcal{M}_1$ w.r.t. β: this gives the output minimal model bisimilar with $\mathcal{M}_1$.

Splitting of a cluster C in a figure

Three clusters: C_1, C_2 and C_3. For some coalition A:



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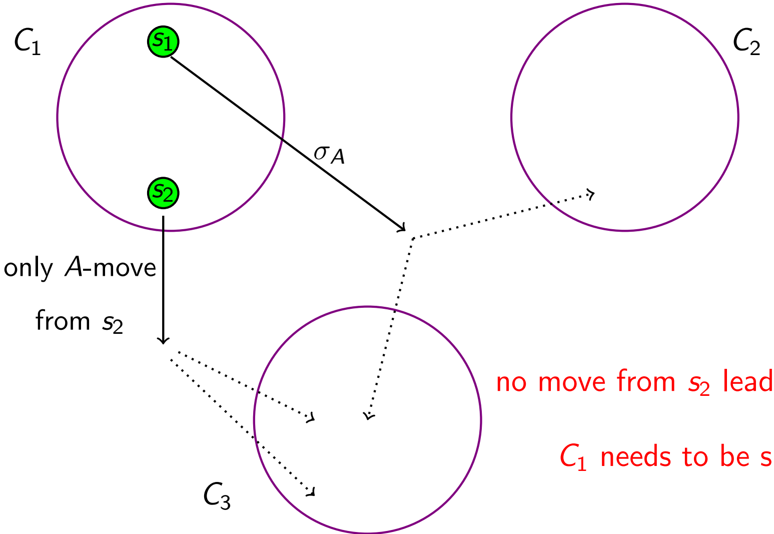

no move from s_2 leads to C_2:

C_1 needs to be split

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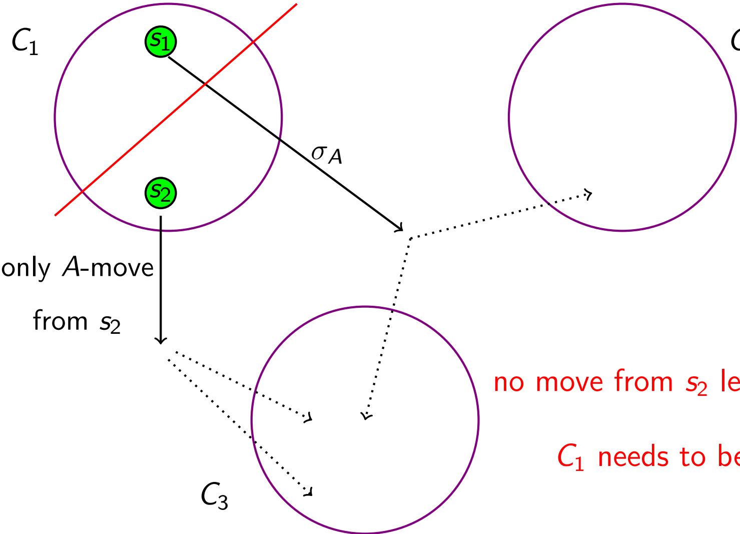


no move from s_2 leads to C_2:

C_1 needs to be split

Properties of the minimization algorithm

- It respects its specification : it computes a model that is minimal w.r.t. bisimulation with the input model $\mathcal{M}_1$;
- It does it in time exponential w.r.t. the number of vertices of $\mathcal{M}_1$ and the number of agents : all the coalitions need to be checked for full bisimilarity!
  NB: when $A \subset A'$, β is an A-alternating bisimulation **does not** imply that β is an A' alternating bisimulation, and β is an A'-alternating bisimulation **does not** imply that β is an A-alternating bisimulation.
- It boils down to computing a maximal fixed point.

  Maximality of the bisimilarity relation β between states of $\mathcal{M}_1$ = minimality of the number of β-equivalences classes of states of $\mathcal{M}_1$ = minimality of the number of states of the output model.

The above properties have been proved

Ongoing and Future Work

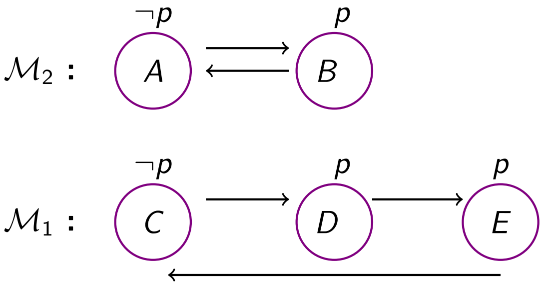
In THIS color, novelties w.r.t. september 2017 (Tableaux conference)

- The algorithm has been implemented and the implementation has been partially tested. Work in progress, TER M1 CILS: Construction of a representative set of tests of the implementation : testing and construction of statistics: automatic;
- Integration to TATL software (Tableaux for ATL), to reduce the size of the models found when the input formula is satisfiable; work in progress, by TER M1 and Amélie David ;
- **TO DO**: Possible to extend our minimization approach to the case of imperfect information ? Look at a recent definition of bisimilarity given by. F. Belardinelli

Ongoing and Future Work

Can we further reduce a model $\mathcal{M}_1$ by considering other aspects? Work in progress with V. Goranko and A. David.

A trivial exemple with just 1 agent and 1 action to illustrate the problem :



The CGM $\mathcal{M}_1$ and $\mathcal{M}_2$ both models $F = \langle 1 \rangle \bigcirc p$ (resp. at state C and A), but: $\mathcal{M}_1$ is not bisimilar to $\mathcal{M}_2$, that is a smaller model of F.

Thanks

MERCI ! QUESTIONS ?