

# GP-SAT: a Framework for Parallel SAT Solving

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- Artificial intelligence
- **Model checking**
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SAT is **NP-complete** [Coo71]

CPU greedy + low memory consumption → parallelization.

My objective is to improve the parallelization of SAT solving.

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## When is $\varphi$ SAT?

- $\varphi = \text{true}$  iff  $\forall \omega \in \varphi, \omega = \text{true}$
- $\omega = \text{true}$  iff  $\exists l \in \omega, l = \text{true}$

# Naive approach

$$\varphi = \omega_1 \wedge \omega_2 \wedge \omega_3 \wedge \omega_4 \wedge \omega_5 \wedge \omega_6$$

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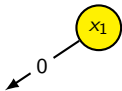
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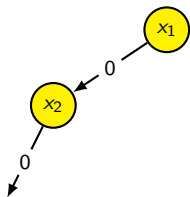
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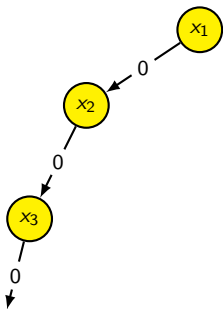
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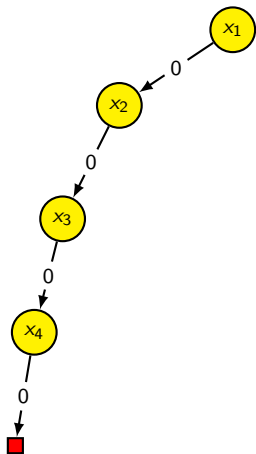
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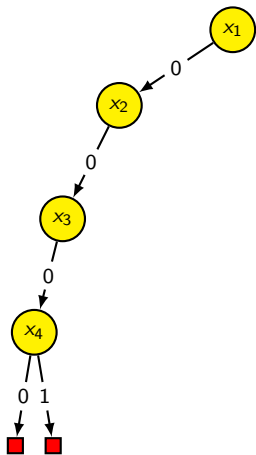
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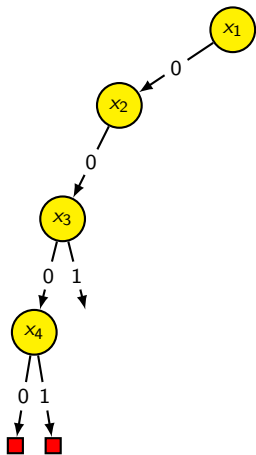


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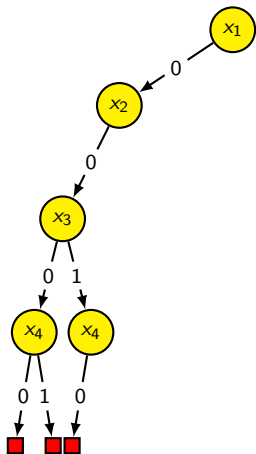
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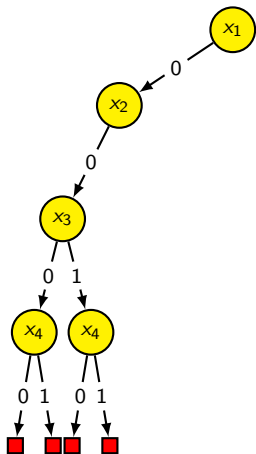
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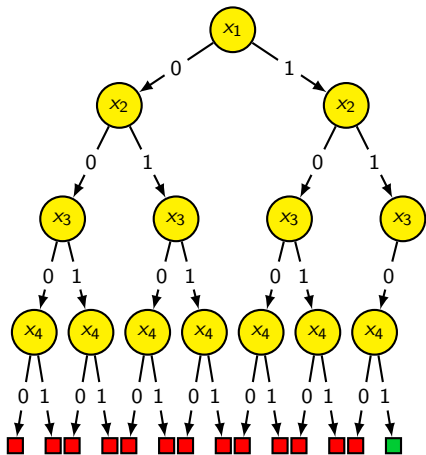
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- Davis, Putnam, Logemann, and Loveland (**DPLL**) [DP60, DLL62]
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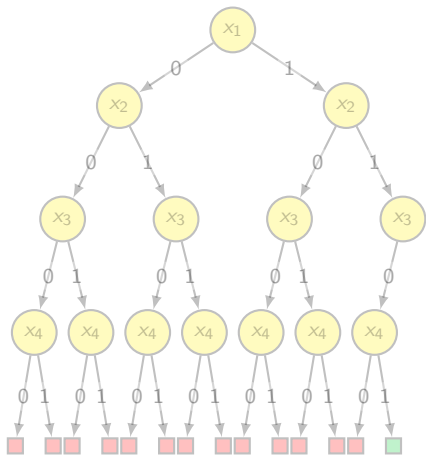
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## Efficiency in practice

- Unit propagation
- Decision and polarity
- Clause learning
- Restart
- Preprocessing

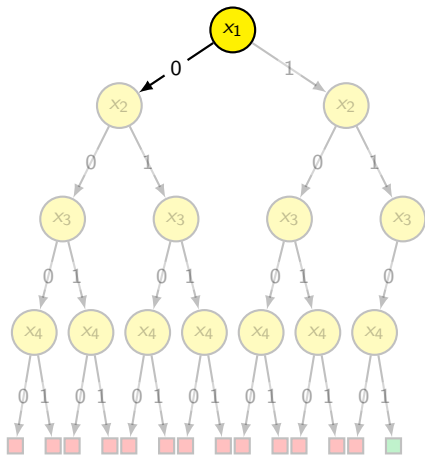
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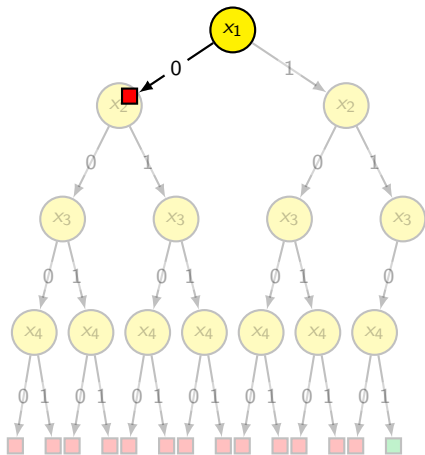


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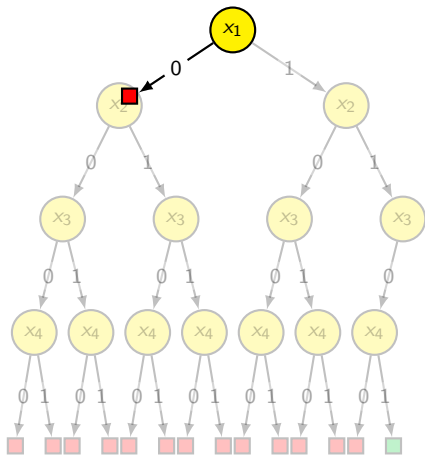
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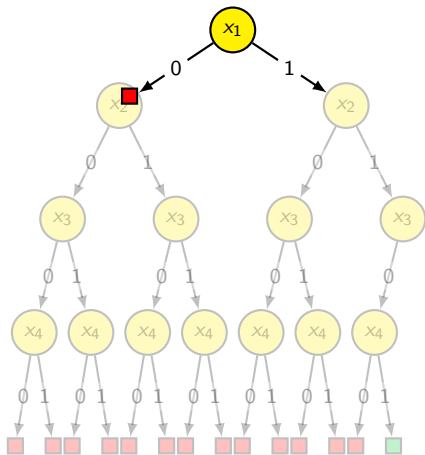
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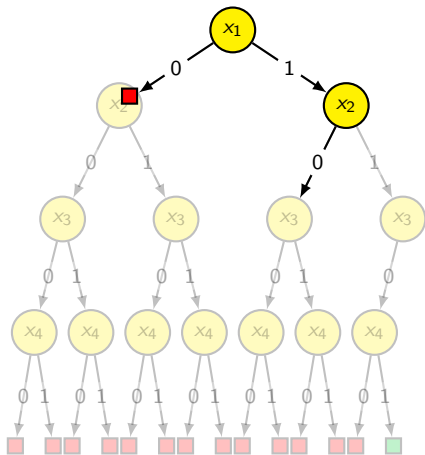
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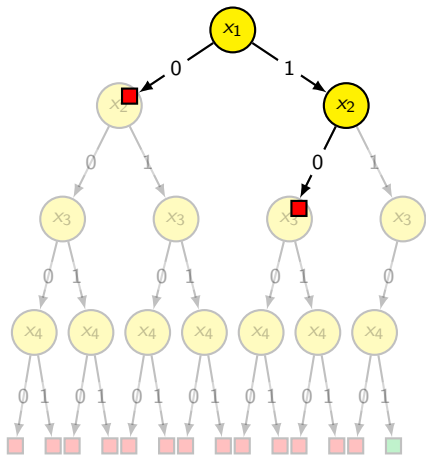
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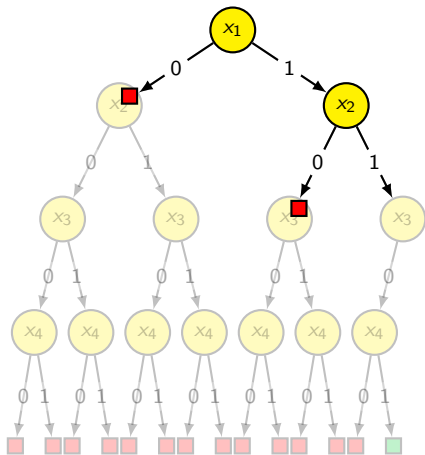
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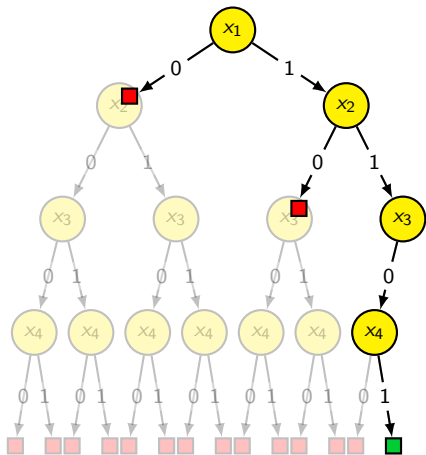
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- **Phase saving** [PD07]:
    - Each assignment is stored
    - The polarity used is the one stored
    - Functions as a cache

# Parallel Resolution

## Motivations

- Bigger and bigger instances
- No more faster CPU
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## SAT Competition 2016

Rank	Name	Type
1	treengeling	DC
2	plingeling	PF
3	cmsat5_run_parallel	PF
4	glucose-syrup	PF
5	cbpenelope	PF
6	csspenelope	PF
7	penelope_48	PF
8	Priss	PF
9	tbParaGlueminisat	PF
10	gluco_par	PF
11	ampharos-sat	DC

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Learnt clauses exchange	possible	possible
Worker search spaces	same	different
Scalable	no*	yes
Easy to implement	yes	no

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Possible contributions:

There also exist **hybrid approaches** (more complex).

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- Adaptive thresholds for these measures [HJS11].

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Which clauses should be exchanged? And between which workers?

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## Problem 2

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## Problem 3

- An implementation → single composition policy
- New heuristic × other mechanisms → difficult

# Example of using GP-SAT

A **generic**, **modular**, and **efficient** framework to build parallel SAT solvers.

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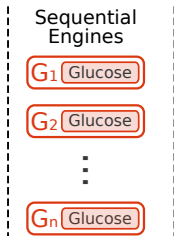
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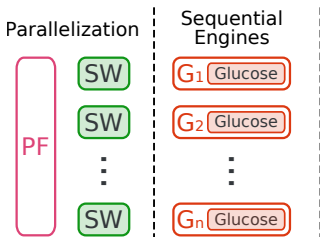
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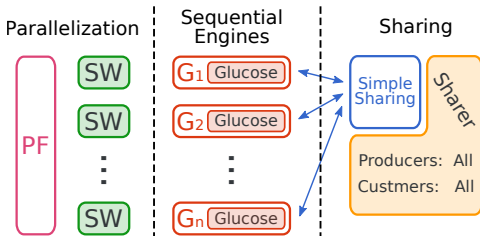
- glucose-syrup [AS14]
  - glucose [AS09]
  - Portfolio



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  - All to all





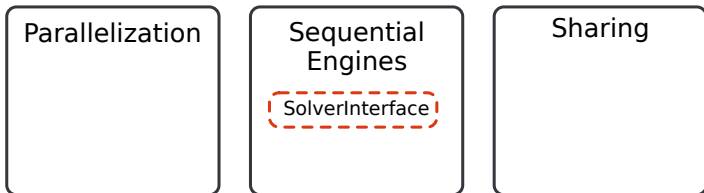
# Architecture of GP-SAT

Parallelization

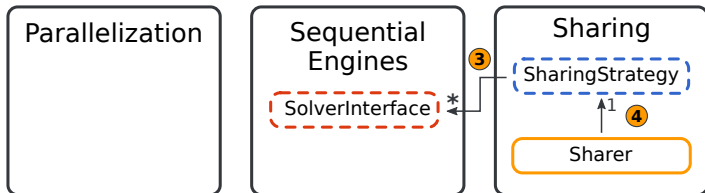
Sequential  
Engines

Sharing

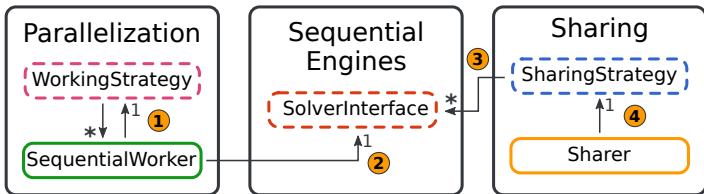
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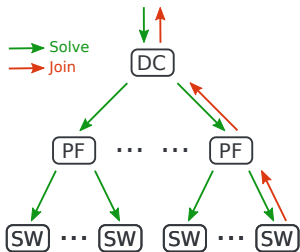
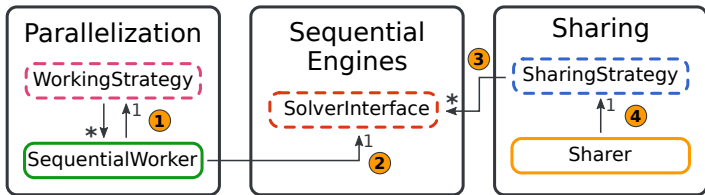
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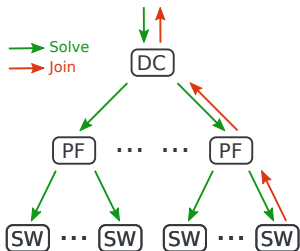
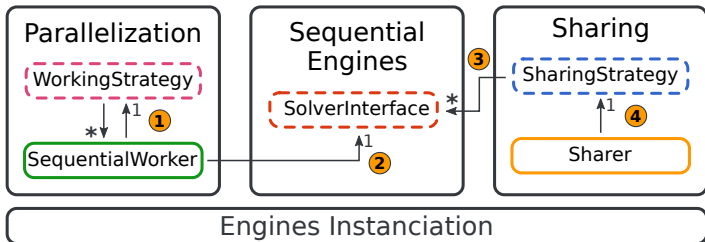


# Architecture of GP-SAT



Exemple of a  
Divide-and-Conquer of  
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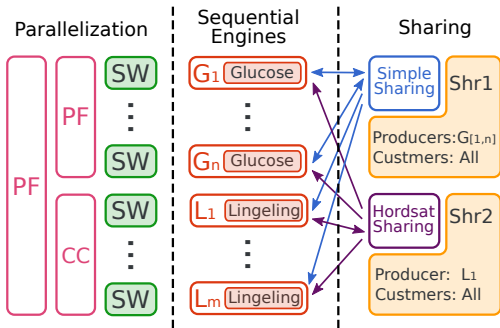
# Others Implementations using GP-SAT

- Hordesat [BSS15]: Lingeling [Bie16] + Portfolio + special sharing.
- Treengeling [Bie16]: Lingeling + Cube-and-Conquer + simple sharing of units.

# Others Implementations using GP-SAT

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Solver mixing all the strategies.





# Experiments & Results

## Parameters

<b>40 processors:</b>	Intel Xeon E7-2860
<b>memory:</b>	500 Go
<b>#cores:</b>	36
<b>#instances:</b>	100 (SAT Race 2015)
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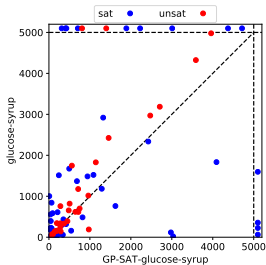
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Solver	All	unsat	sat
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Dots over the line are good ones.

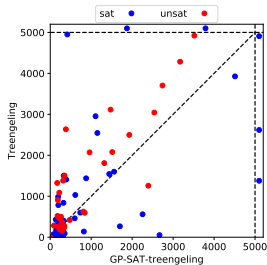
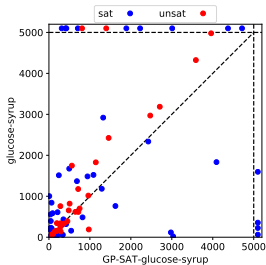
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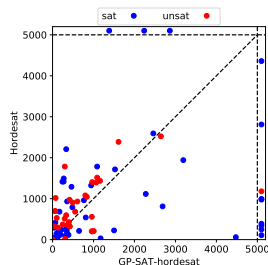
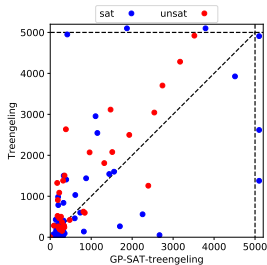
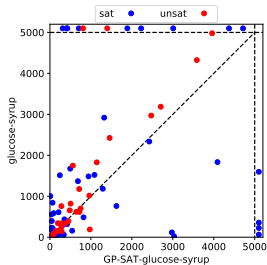
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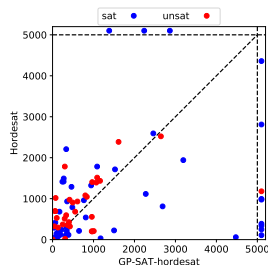
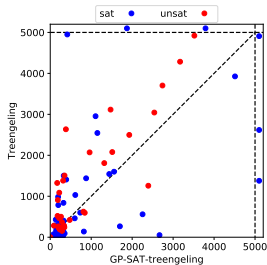
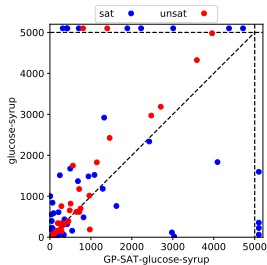
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- GP-SAT is **modular** and **generic**.
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- GP-SAT is **modular** and **generic**.
- Experiments shows that GP-SAT is **efficient**.
- **The proof of concept is validated**: good results for a prototype.
  
- Bench with more instances (SAT Competition 2016).
- Toward a more generic architecture.
- **Implementing more strategies to test the architecture.**
- **Participating to the SAT Competition 2017.**
- **Use GP-SAT for our own contributions.**

Thank you!





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# Clause Learning

Let  $x_1 = 0$ ,  $x_4 = 0$ ,  $x_8 = 1$  be the reasons of a conflict.

$$\varphi \equiv \varphi \wedge \omega_{\text{learnt}}, \text{ where } \omega_{\text{learnt}} = \{x_1, x_4, \neg x_8\}$$

## Problem

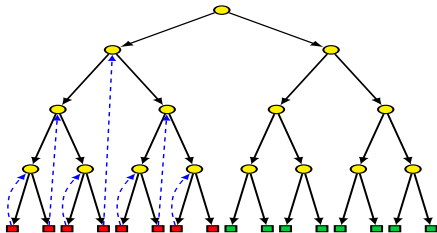
Too many clauses are learnt  
→ not all of them can be kept.

### Selection criteria

- Activity: keep clauses used for propagation/conflict resolution.
- Size: longer clauses are less used for propagation.
- **Literal Block Distance (LBD)** [AS09]:
  - number of decision level gathered by a clause.

# Restart

## Motivations



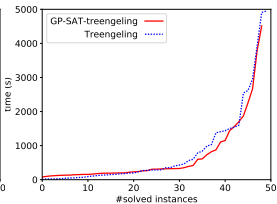
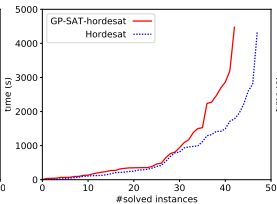
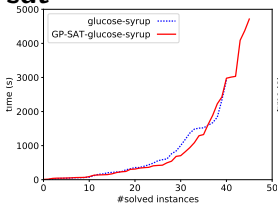
Escape from hard zones  
(heavy-tailed) [BHvM09].

## Strategies

- #conflicts: arithmetic, geometric, Luby progressions.
- Progress saving: prevents from softness.
- **Average LBD**: are we in a good region for learning?
- **SAT/UNSAT**: are we close to find a solution?

# Cactus Plots

sat



unsat

