

TiAMo: Timed Automata Model-checker

P. Bouyer, M. Colange, N. Markey, O. Sankur

April 8, 2016

Motivation (1)

Timed systems

- Models for real-time systems (transportation systems, production lines, networks . . .)
- Need for verification
 - is a task realizable under given real-time constraints?
 - how fast can a task be?
 - . . .

Motivation (2)

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Discrete time is not enough:

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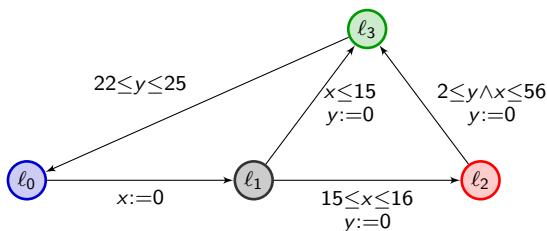
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TiAMo

- a platform for experiments
- assert new algorithms
- compare algorithms

Currently focused on (weighted) reachability

Timed Automata [AD94]



$(l_0, x = 0, y = 0)$	\Rightarrow delay 0.32	$(l_0, x = 0.32, y = 0.32)$
$\Rightarrow (l_1, x = 0, y = 0.32)$	\Rightarrow delay 2.64	$(l_1, x = 2.64, y = 2.96)$
$\Rightarrow (l_3, x = 2.64, y = 0)$	\Rightarrow delay 23.13	$(l_1, x = 25.77, y = 23.13)$
$\Rightarrow (l_0, x = 25.77, y = 23.13)$	$\Rightarrow \dots$	

Timed Automata formally

Definition

- Q finite set of locations (or discrete states)
- X finite set of clocks
- $T \subseteq Q \times \mathcal{G} \times 2^X \times Q$ where
 - \mathcal{G} set of guards, i.e. conjunctions of comparisons ($<, \leq, =, \geq, >$) of a clock with a constant integer

$v \in \mathbb{R}_{\geq 0}^X$: clock valuation

$(\ell, v) \in Q \times \mathbb{R}_{\geq 0}^X$: (extended) state

2 flavors of transitions

- time elapse: delay all clocks by $\delta > 0$
- discrete transition

Reachability Analysis

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Is a given location reachable?

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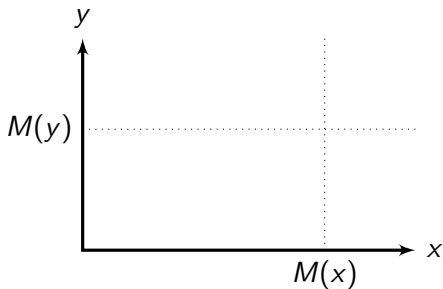
- uncountably infinite
- uncountably infinitely branching

Regions (1)

For a clock x , $M(x) = \max\{k \mid x \bowtie k \text{ in the automaton}\}$

Inactive clock

If $x > M(x)$, its exact value no longer matters: *inactive clock*.



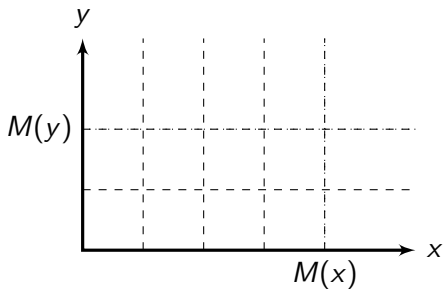
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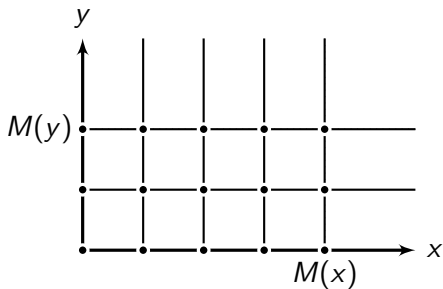
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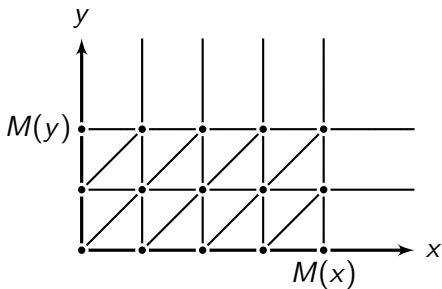
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- comparison with integers
- strict/large inequalities
- time elapse



Regions (2)

Time-abstract bisimulation

If v and v' are in the same region, then same locations reachable from (ℓ, v) and (ℓ, v') .

Reachability is decidable [AD94]

- ℓ reachable iff ℓ reachable in the region graph
- the region graph is *finite*

At most $(2M + 2)^{|X|} 2^{|X|} |X|!$ regions
 \Rightarrow combinatorial explosion

The general algorithm

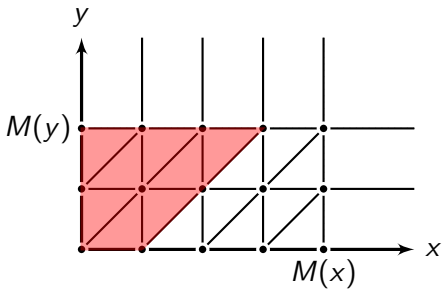
Algorithm 1: Symbolic algorithm for reachability

```
1 PASSED  $\leftarrow \emptyset$ 
2 WAITING  $\leftarrow \{(\ell_0, Z_0)\}$ 
3 while WAITING  $\neq \emptyset$  do
4   | select  $(\ell, Z)$  from WAITING
5   | if  $\ell \in \text{Goal}$  then
6   |   | return Reachable
7   | if  $(\ell, Z)$  has not been explored yet then
8   |   | add  $(\ell, Z)$  to PASSED
9   |   | add  $\text{Post}(\ell, Z)$  to WAITING
10 return Not reachable
```

Zones (1)

The regions are too small.

$$(0 \leq x) \wedge (0 \leq y) \wedge (x \leq y - 1) \wedge (y \leq 2)$$



Zones (2)

Zone [Dil89]

Convex set of valuations, intended to capture many regions at once

- regions are zones
- nice algorithmic representation and manipulation (DBM)
 - the successor of a zone is a zone
 - polynomial-time algorithm for resets, time elapse, guard evaluation, inclusion

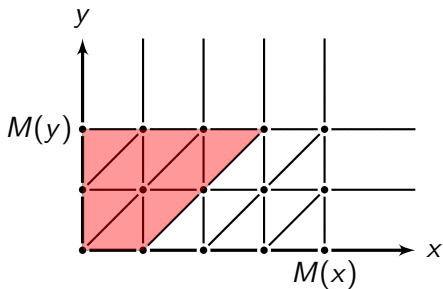
The zone graph is still correct and sound for reachability properties. Implementations use zones (as DBM) instead of regions.

The problem with zones

Drawback of zones

convexity hinders proper representation of inactive clocks

Let the time elapse in the **red zone**.



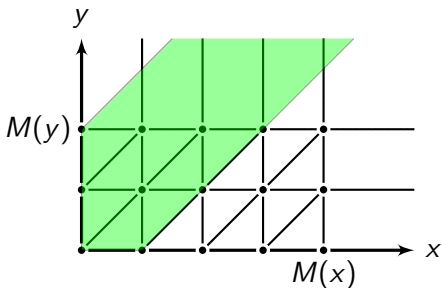
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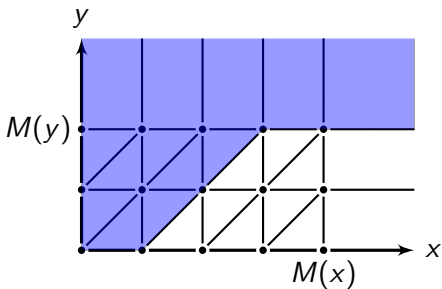
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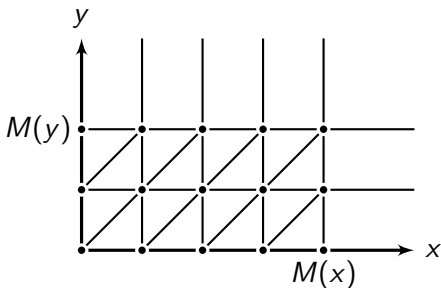
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Problem: the zone graph is no longer finite (!)

Termination: abstractions and comparisons

zone abstraction [DT98]

Enlarge zones, in an attempt to guarantee termination

- try to capture entire regions

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- plain set inclusion
- inclusion of abstractions [HKSW11, HSW12]: $Z \subseteq \text{abstr}(Z')$
 - abstraction is not explicitly build
 - same asymptotic complexity as set inclusion (on DBM)

The parameterized algorithm

Algorithm 2: Symbolic zone algorithm for reachability

Parameter: α // Abstraction operator

Parameter: \preceq // Zone comparison

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7   | if for all  $(\ell, Z') \in \text{PASSED}$ ,  $Z \not\preceq Z'$  then
8   |   | add  $(\ell, \alpha(Z))$  to PASSED
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Termination

Termination ensured by an appropriate **combination** of

- abstraction
- comparison
- (sometimes) restriction on the input models, e.g. all clocks are upper-bounded

(usually) termination \Leftarrow comparison is well-founded over the set of abstracted zones

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Usually, trade-off between

- computational complexity of abstraction/comparison
- number of zones explored (hence number of abstractions/comparisons performed)

Algorithmic bottleneck

Main bottleneck (from experimental observations)

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- merge PASSED and WAITING [BBD⁺02]
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- merge PASSED and WAITING [BBD⁺02]
already explored states are NOT added to WAITING
- order of exploration matters:
BFS (larger zones) *often* better than DFS (smaller zones)
- smart exploration [HT15]:
if s subsumes s' , the successors of s subsume those of s'
if s subsumes s' , remove s' and its successors from WAITING

Reachability with weights

Add non-negative weights to the timed automata model

- spends t time in location ℓ : weight $t \times \text{weight}(\ell)$
- take edge e : weight $\text{weight}(e)$

Optimal reachability

Given a location, what is the smallest weight to reach it?

Similar algorithms and problems:

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- abstraction/comparisons [BCM16]

To sum up

A symbolic algorithm for reachability analysis, parameterized by:

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Non-trivial interaction between parameters

Efficiency results from a trade-off between parameters

A new tool, why?

Many tools out there

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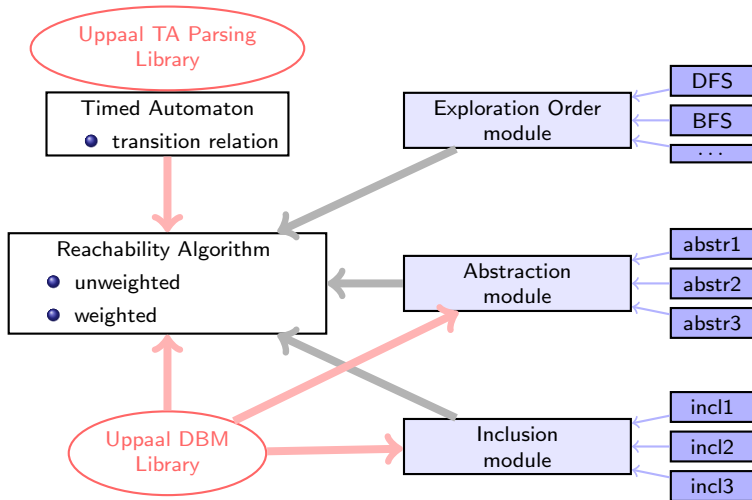
- most of them are no longer active

A new tool, why?

Many tools out there

- most of them are no longer active
- source code access problems
 - it is not always clear what is exactly implemented
 - not suitable for rigorous comparisons (change one parameter at a time)

TiAMo architecture



What is implemented

Exploration strategies

- BFS, DFS
- best cost first (for weighted models)
- preference-based (use a special “preference” variable in the model)
- “smart”: approximate implementation of [HT15]

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Inclusions

- set inclusion
- abstract inclusion [HKS11, HSW12]
- weighted set inclusion [RLS06]
- abstract weighted inclusion [BCM16]

A concrete application

We designed a new abstract inclusion for weighted zones [BCM16]

Experimental comparison with state-of-the-art algorithm with TiAMo

- various metrics
 - wall-clock time
 - number of symbolic states explored
 - number of comparisons done
- in various contexts
 - with different exploration strategies

What's next?

Towards Games

Reachability games (both weighted and unweighted)

- controller synthesis

Models Database

- collect models
- format issues

Thank you!

<https://git.lsv.fr/colange/tiamo>

Licensed under GPL

Any questions?

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