Intrinsically Live Structure and Deadlock Control in Generalized Petri Nets Modeling Flexible Manufacturing Systems

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Background

Petri nets

A Petri net is a graphical tool for the description and analysis of concurrent process...[1]

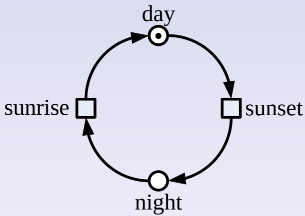


Figure: The circle of life.

[1] C. A. Petri and W. Reisig, "Petri net," *Scholarpedia*, 3(4):6477, 2008.

[2] M. Silva, "50 years after the PhD thesis of Carl Adam Petri: A perspective," in *Preprints of the 11th International Workshop on Discrete Event Systems*, pp. 13–20, Guadalajara, Mexico, October 3rd - 5th, 2012.

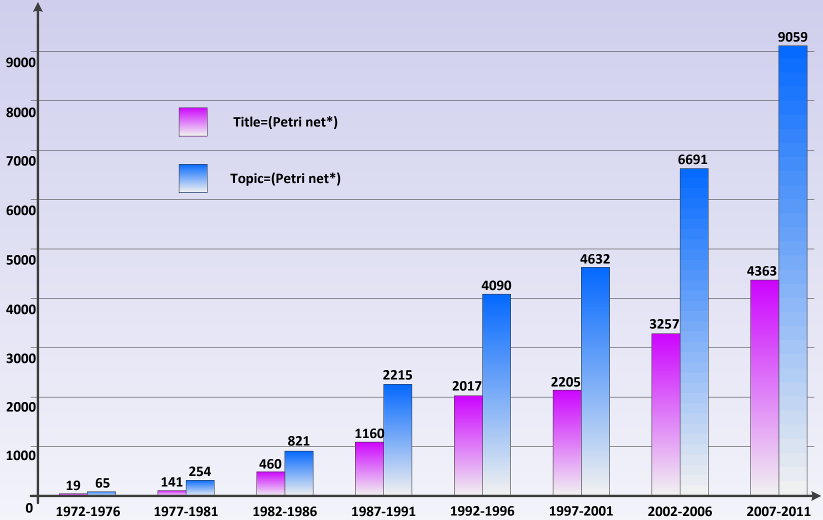


Figure: A histogram of the numbers of SCI papers on Petri nets in the past 40 years.

Figure: A deadlock between two oxen. (Picture from Internet)

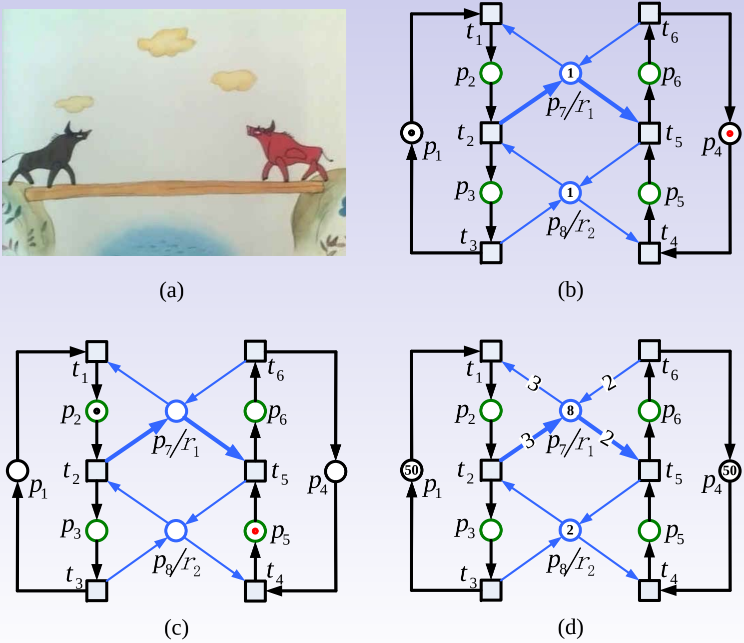


Figure: Oxen and Petri nets.

Motivation

What we face:

- Siphon structures do not consider weight information;
- How can we extend siphon-based methods to control generalized Petri nets;
- Controlled generalized Petri nets with Siphon-based methods limit strongly the system behavior.

What we want:

- A new kind of structural objects tied with deadlock-freedom and liveness;
- A new policy for deadlock-control/liveness-enforcement.

Main Work

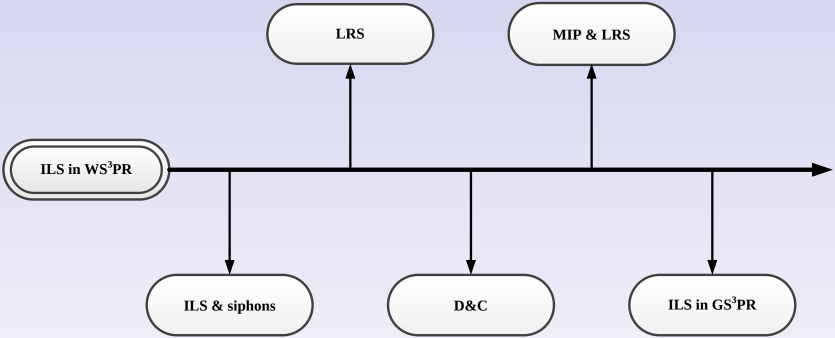


Figure: All work.

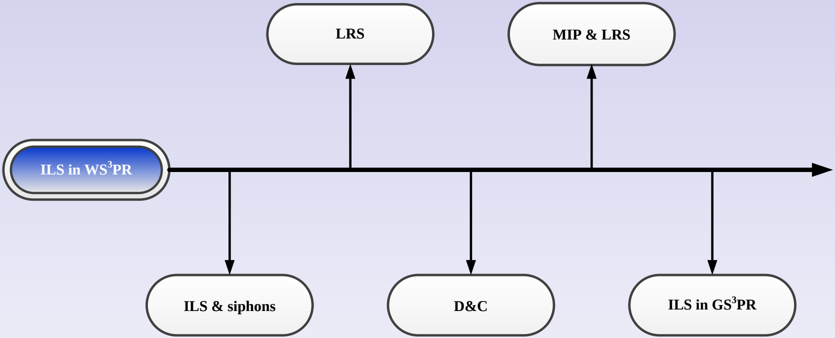


Figure: Intrinsically live structure.

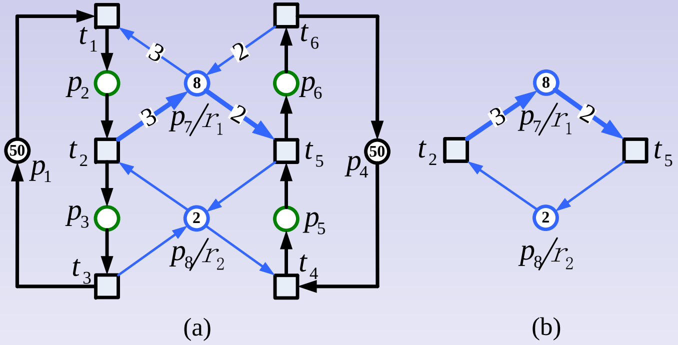


Figure: A WS^{3}PR and its WSDC.

- A structural object carrying weight information;
- A structure intuitively reflecting circular waits;
- A numerical relationship between initial marking and arc weights.

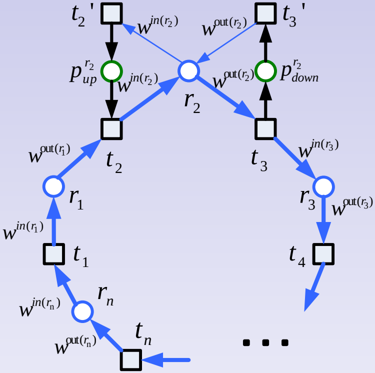


Figure: A WSDC.

- A subnet consisting of places, transitions, their arcs, and weights;
- A competition path $t_2 r_2 t_3$;
- The upstream activity place $p_{up}^{r_2}$ and downstream one $p_{down}^{r_2}$ compete against each other.

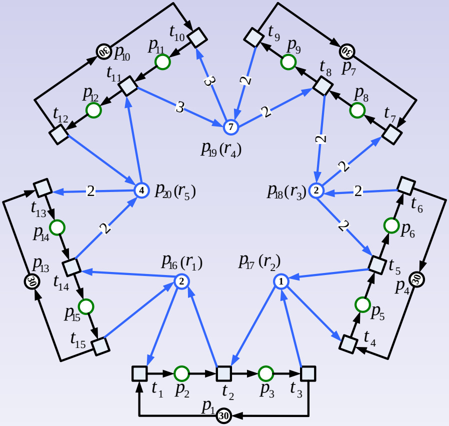


Figure: A revised dining philosopher problem modeled by WS^{3}PR with a unique WSDC.

Main results:

Restriction 1

Given a resource place r, it satisfies the following two conditions:
(1) $(M_0(r) \ mod \ w_i^{in(r)}) \geq w_i^{out(r)}$, where $M_0(r)$ is the initial marking of r, and $w_i^{in(r)}$ (resp. $w_i^{out(r)}$) is the in-arc (resp. out-arc) weight of the ith competition path in $\mathbf{L}_W$; and
(2) There exists no such an $n-$dimensional row vector $\mathbf{A} = [a_1 \ \ldots \ a_i \ \ldots \ a_n]$ such that $0 \leq (M_0(r) - \mathbf{A}[\mathbf{W}^{in(r)}]^T) < \min\{w_i^{out(r)}\}$, where $a_i \in \mathbb{N}$.

Theorem 1

A marked WS^{3}PR (N, M_0) is live if every WSDC satisfies Restriction 1.

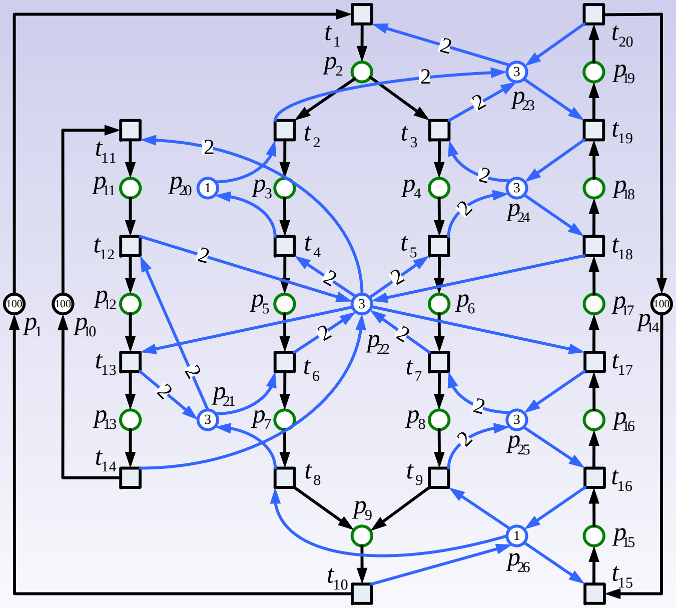


Figure: A Live WS^{3}PR with all WSDCs satisfying Restriction 1.

The meaning of the work:

- A new method to prevent deadlocks;
- Need no external controller;
- Enforcing liveness by increasing or decreasing the numbers of resources.

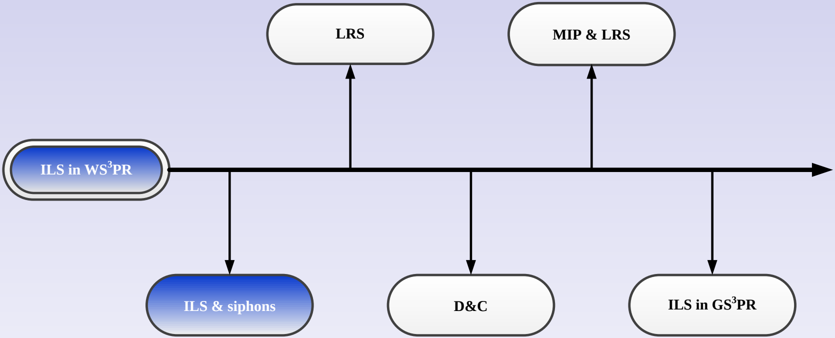


Figure: ILS and siphon.

Main results:

Theorem 2

A siphon S in a marked WS^3PR is never insufficiently marked if all WSDCs contained in it satisfy Restriction 1.

Theorem 3

A siphon in a marked WS^3PR is minimally controlled if all WSDCs contained in it satisfy Restriction 1.

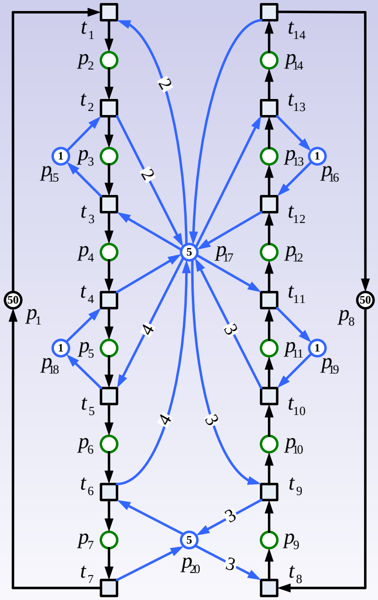


Figure: A WS^{3}PR with 6 resource places.

A set of elementary siphons chosen from all 31 strict minimal ones need to be controlled:

$\overline{S}_{16} = 4p_6 + 3p_9$, $M_0(V_{S16}) = 10 - 6 = 4$;

$\overline{S}_{24} = p_4 + p_5$, $M_0(V_{S24}) = 6 - 4 = 2$;

$\overline{S}_{28} = 2p_2 + p_3$, $M_0(V_{S28}) = 6 - 4 = 2$;

$\overline{S}_{29} = 3p_{10} + p_{11} + p_{12} + p_{13}$, $M_0(V_{S29}) = 7 - 4 = 3$;

$\overline{S}_{30} = 3p_{10} + p_{11}$, $M_0(V_{S30}) = 6 - 3 = 3$; and

$\overline{S}_{31} = p_{12} + p_{13}$, $M_0(V_{S31}) = 6 - 4 = 2$.

By checking all WSDCs in every elementary siphon, the elementary siphons really need to be controlled:

~~$\overline{S}_{16} = 4p_6 + 3p_9$, $M_0(V_{S16}) = 10 - 6 = 4$~~;

$\overline{S}_{24} = p_4 + p_5$, $M_0(V_{S24}) = 6 - 4 = 2$;

~~$\overline{S}_{28} = 2p_2 + p_3$, $M_0(V_{S28}) = 6 - 4 = 2$~~;

$\overline{S}_{29} = 3p_{10} + p_{11} + p_{12} + p_{13}$, $M_0(V_{S29}) = 7 - 4 = 3$;

~~$\overline{S}_{30} = 3p_{10} + p_{11}$, $M_0(V_{S30}) = 6 - 3 = 3$~~; and

$\overline{S}_{31} = p_{12} + p_{13}$, $M_0(V_{S31}) = 6 - 4 = 2$.

The meaning of the work:

- Enlarge the application scope of ILS-based method;
- Improve the siphon-based methods;
- Obtain more reachable states with less control costs.

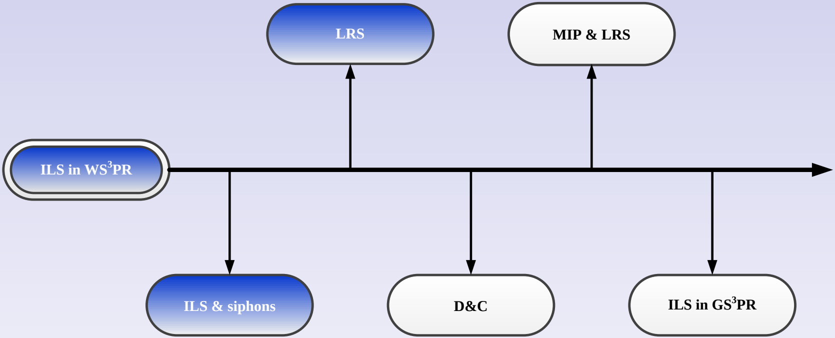


Figure: Liveness and ratio-enforcing supervisor.

Basic ideas:

- Impose a well-designed supervisor with intrinsically live structures to break the chain of circular waits;
- Consider the resource usage ratios of upstream and downstream activity places and the relation between them.

$$\lambda^{r \rightarrow p_{up}} = \frac{M(p_{up}) \cdot w^{in(r)}}{M_0(r)}$$

$$\lambda^{r \rightarrow p_{down}} = \frac{M(p_{down}) \cdot w^{out(r)}}{M_0(r)}$$

$$\lambda_i^{r \rightarrow p} = \frac{(\lfloor \frac{M_0(r)}{W(r,t)} \rfloor - i) \cdot W(r,t)}{M_0(r)},\ i \in \{0, 1, 2, \ldots, \lfloor \frac{M_0(r)}{W(r,t)} \rfloor\}$$

Restriction 2

Given WS^{3}PR (N, M_0), a resource place r satisfies the following two conditions:

(1) $1 - max\lambda^{r \to p_{upi}} \geq min\lambda^{r \to p_{downi}}$; and

(2) $1 - \sum_{i=1}^{n} \lambda^{r \to p_{upi}} \geq min\{min\lambda^{r \to p_{downi}}\}$, if $1 - \sum_{i=1}^{n} \lambda^{r \to p_{upi}} > 0$;

where p_{upi} and p_{downi} are upstream and downstream activity places with respect to every competition path containing r,

$\lambda^{r \to p_{upi}} \in [max\lambda^{r \to p_{upi}},\ 0]^{\lambda^{r \to p_{upi}}}$,

$\lambda^{r \to p_{downi}} \in [max\lambda^{r \to p_{downi}},\ 0]^{\lambda^{r \to p_{downi}}}$, and n is the number of columns of weight matrix representing n competition paths with the same resource place r.

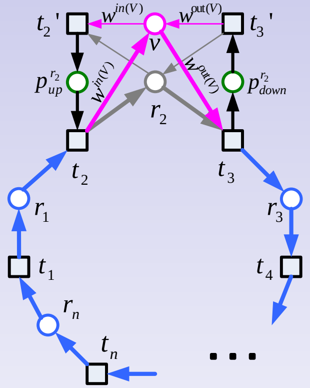


Figure: A WSDC with an LRS monitor.

- Design a control path satisfying Restriction 2;
- Impose the control path to a competition one;
- Manipulate the resource allocation.

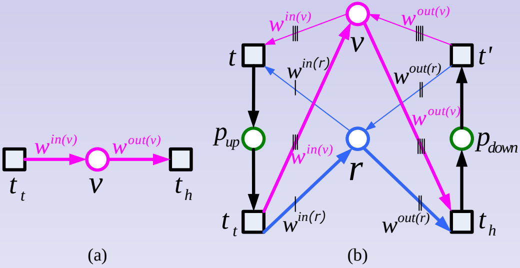


Figure: A control path and an LRS monitor.

$$
\begin{cases}
\min M_0(v) + w^{in(v)} \\
s.\ t. \\
M_0(v) - w^{in(v)} \cdot \lfloor \dfrac{M_0(r) \cdot (1 - min\lambda^{r \to p_{down}})}{w^{in(r)}} \rfloor \geq w^{out(v)} \\
M_0(v) \geq 1 \\
w^{in(v)} \geq w^{out(v)} + 1 \\
w^{out(v)} \geq 1
\end{cases}
$$

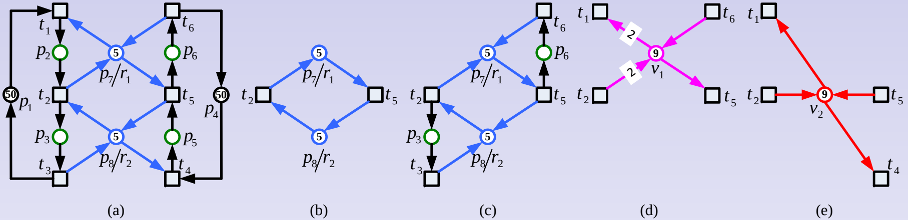


Figure: LRS and siphon-monitor.

- The basic ideas are different;
- The structural objects are different;
- The size of supervisors are different;
- Resource usage ratio and parameters.

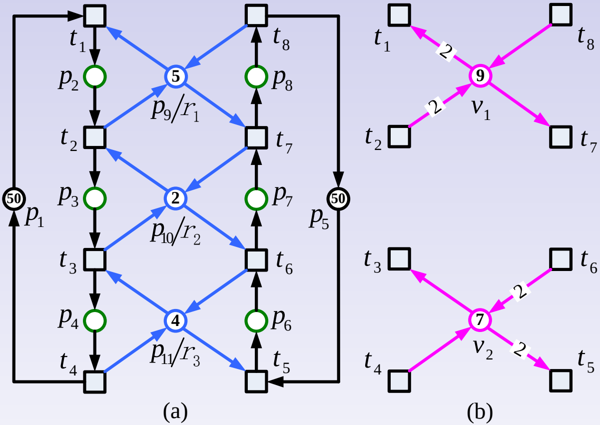


Figure: An S^3PR net model and its LRS monitors.

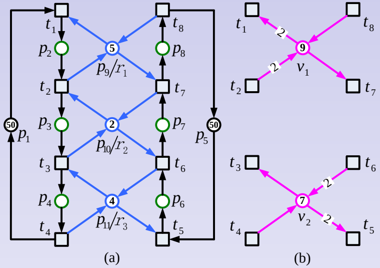


Table: A case study of the S^3PR net with LRS monitors

case (i)	$M_0(P_R)$	$M_0(V)$	$w^{in(v)}$	$w^{out(v)}$	states	percentage
0	$5r_1 + 2r_2 + 4r_3$	/	/	/	1853	/
1	$5r_1 + 2r_2 + 4r_3$	$9v_1 + 7v_2$	$w^{in(v_1)} = 2$ $w^{in(v_2)} = 2$	$w^{out(v_1)} = 1$ $w^{out(v_2)} = 1$	1645	100%
2	$4r_1 + 2r_2 + 4r_3$	$7v_1 + 7v_2$	$w^{in(v_1)} = 2$ $w^{in(v_2)} = 2$	$w^{out(v_1)} = 1$ $w^{out(v_2)} = 1$	1147	69.73%
3	$3r_1 + 2r_2 + 4r_3$	$5v_1 + 7v_2$	$w^{in(v_1)} = 2$ $w^{in(v_2)} = 2$	$w^{out(v_1)} = 1$ $w^{out(v_2)} = 1$	732	44.50%
4	$2r_1 + 2r_2 + 4r_3$	$3v_1 + 7v_2$	$w^{in(v_1)} = 2$ $w^{in(v_2)} = 2$	$w^{out(v_1)} = 1$ $w^{out(v_2)} = 1$	400	24.32%
5	$5r_1 + 2r_2 + 4r_3$	$9v_1 + 8v_2$	$w^{in(v_1)} = 2$ $w^{in(v_2)} = 3$	$w^{out(v_1)} = 1$ $w^{out(v_2)} = 1$	1407	85.53%

The meaning of the work:

- The size of an LRS is bounded by the number of resource places;
- No new problematic structures generated;
- Parameterized controller;
- Intuitive and easy to understand.

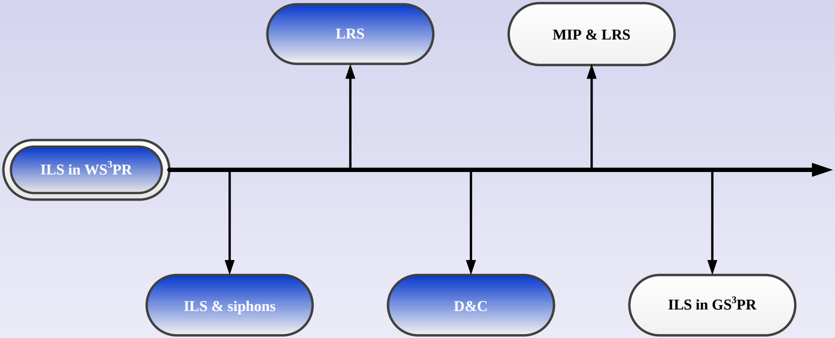


Figure: Divide-and-Conquer.

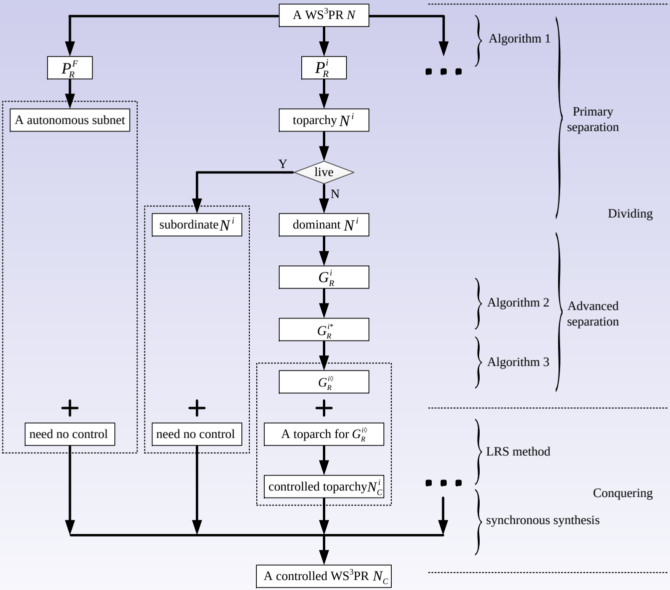


Figure: A schematic diagram of D&C.

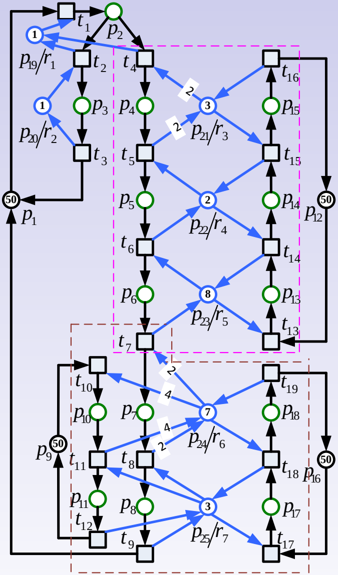


Figure: Control subnets.

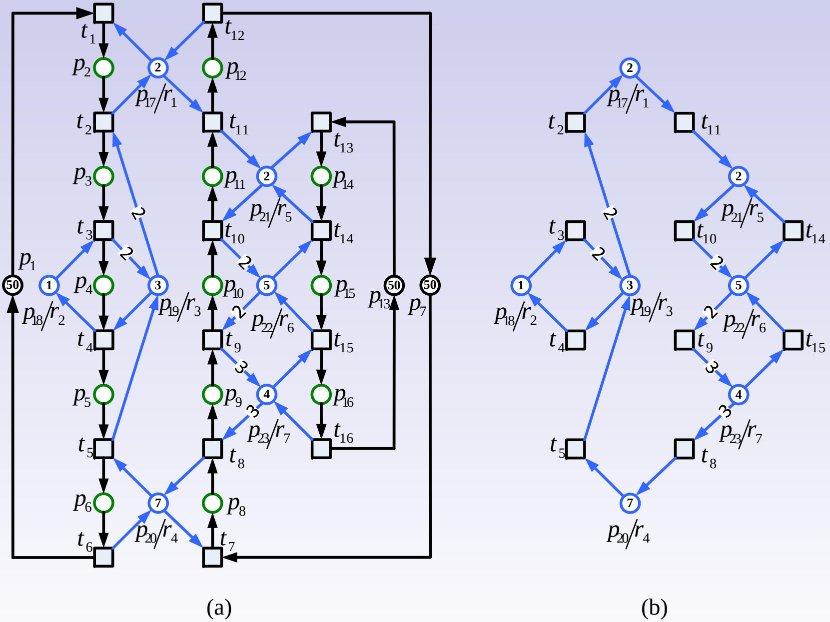


Figure: A unique subnet.

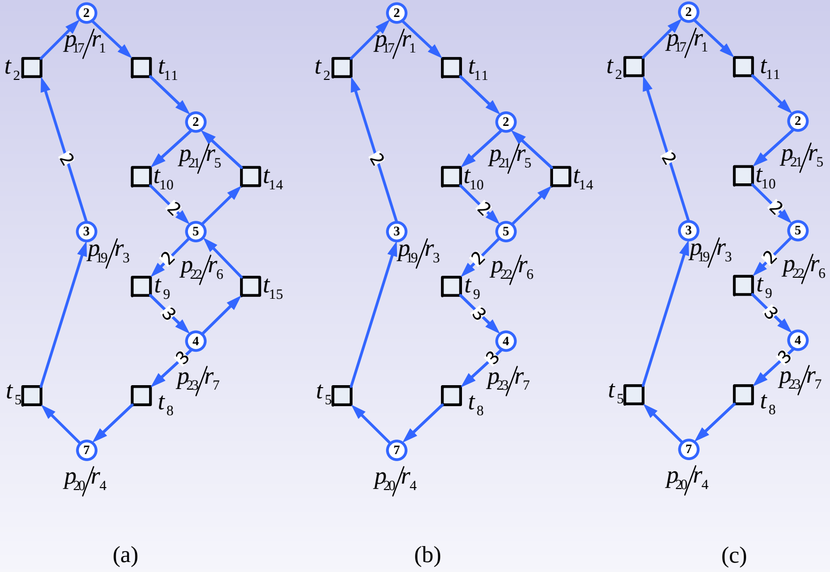
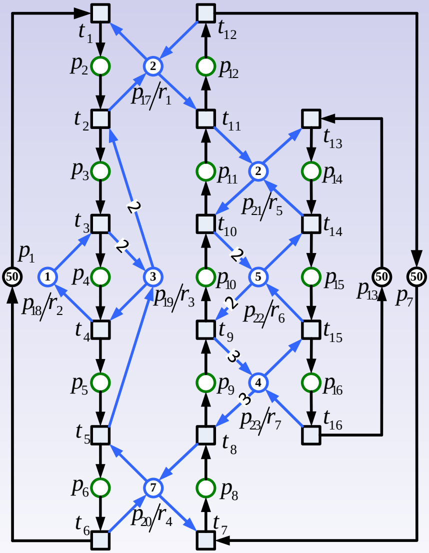


Figure: Decomposition of a subnet.


t1
t2
t3
t4
t5
t6
t7
t8
t9
t10
t11
t12
t13
t14
t15
t16
p1
p2
p3
p4
p5
p6
p7
p8
p9
p10
p11
p12
p13
p14
p15
p16
p17/r1
p18/r2
p19/r3
p20/r4
p21/r5
p22/r6
p23/r7
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50
50
1
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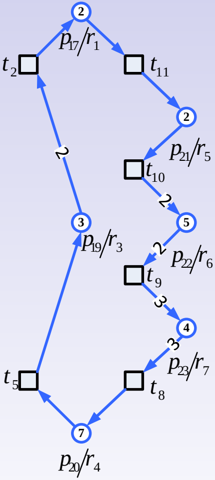
3
7
2
5
4
2
2
2
2
3
3

(a)


t2
t11
t10
t9
t8
t5
p17/r1
p21/r5
p19/r3
p22/r6
p23/r7
p20/r4
2
2
3
5
4
7
2
2
2
3
3

(c)

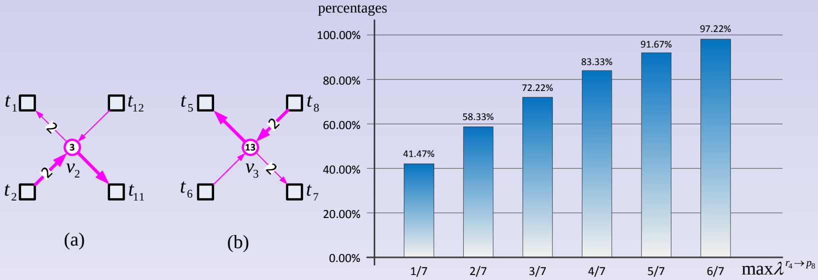


Table: Different control effects of the LRS monitor v_3 with different control parameters.

admissible range	$\lambda^{r_4 \to p_8}$	$M_0(v_3)$	$w^{in(v_3)}$	$w^{out(v_3)}$	all reachable states	percentage	live
$[1, 0]^{r_4 \to p_8}$	1	/	/	/	933, 112	100%	No
$[6/7, 0]^{r_4 \to p_8}$	6/7	13	2	1	907, 192	97.22%	Yes
$[5/7, 0]^{r_4 \to p_8}$	5/7	17	3	1	855, 352	91.67%	Yes
$[4/7, 0]^{r_4 \to p_8}$	4/7	19	4	1	777, 592	83.33%	Yes
$[3/7, 0]^{r_4 \to p_8}$	3/7	19	5	1	673, 912	72.22%	Yes
$[2/7, 0]^{r_4 \to p_8}$	2/7	17	6	1	544, 312	58.33%	Yes
$[1/7, 0]^{r_4 \to p_8}$	1/7	13	7	1	388, 792	41.47%	Yes

The meaning of the work:

- Analyze and control large-size WS^3PR net models;
- Precisely locate and control the genuine structures contributing to non-liveness;
- Reduce control cost with simple and parameterized controllers.

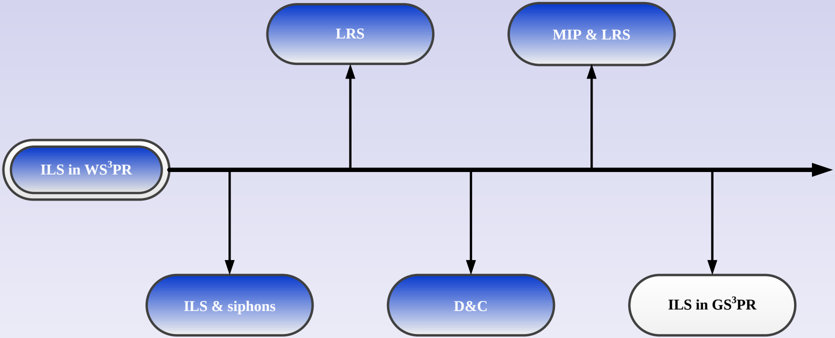


Figure: MIP and LRS.

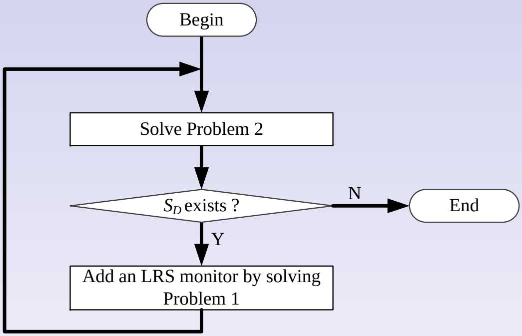


Figure: A schematic diagram of the iterative method.

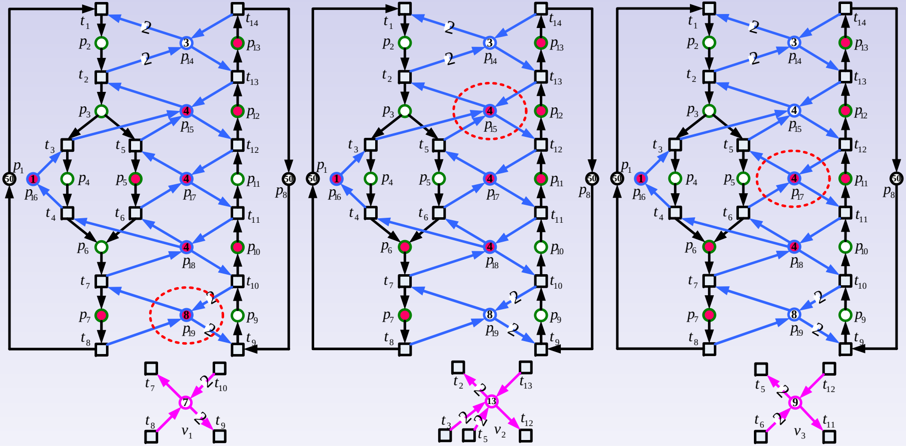


Figure: An iterative control of a WS^{3}PR net model.

Table: Details of the iterative control of the WS^3PR net model.

MIM siphon obtained by MIP	r_i	v_i	reachable states	dead states	live transitions
/	/	/	3, 334, 653	30	0
$\{p_5, p_7, p_{10}, p_{12}, p_{13}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}\}$	p_{19}	v_1	2, 663, 888	6	0
$\{p_6, p_7, p_{11}, p_{12}, p_{13}, p_{15}, p_{16}, p_{17}, p_{18}\}$	p_{15}	v_2	2, 613, 824	1	0
$\{p_6, p_7, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}\}$	p_{17}	v_3	2, 500, 037	0	14

The meaning of the work:

- Avoid the enumeration of all WSDCs;
- Improve the computational efficiency;
- The number of iterations is bounded by that of resource places.

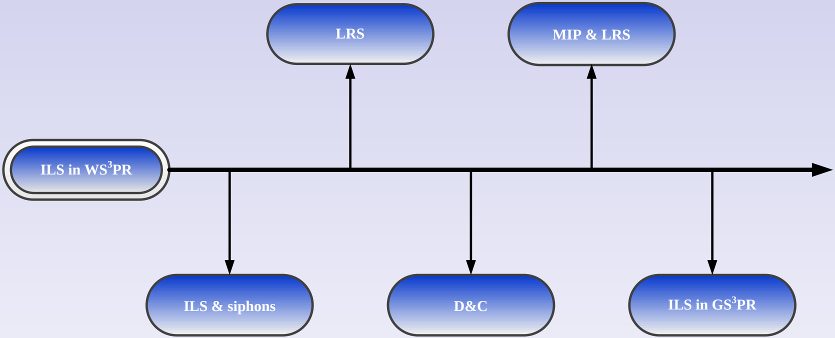


Figure: ILS in GS^3PR.

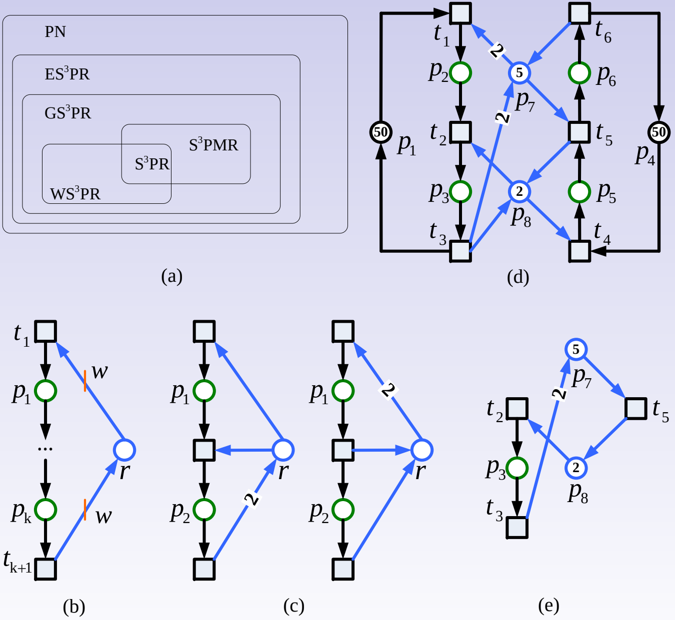


Figure: GS³PR and ILS.

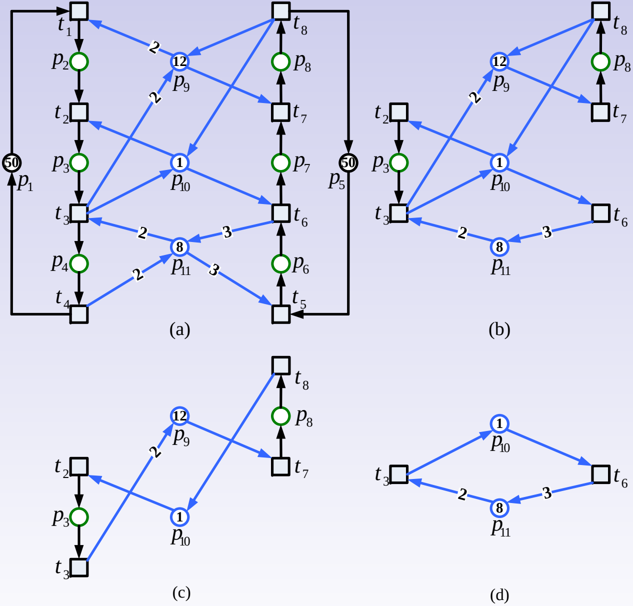


Figure: GS^3PR and extended WSDCs.

Table: A Comparison between the control effects of the elementary siphons-based method (ES) and the intrinsically live structures-based one (ILS) in this work for the GS^3PR net model.

Description	$M_0(P_R)$	$M_0(V)$	Arcs and weights of monitors	$\lvert R(N, M_0)\rvert$	Live
Original net: without control	$12p_9 + p_{10} + 8p_{11}$	/	/	234	No
ES by connecting output arcs of monitors to source transitions	$12p_9 + p_{10} + 8p_{11}$	$11v_1 + 5v_2$	$W(v_1, t_1) = W(t_2, v_1) = 2$, $W(v_1, t_5) = W(t_7, v_1) = 1$, $W(v_2, t_1) = W(t_3, v_2) = 1$, $W(v_2, t_5) = W(t_6, v_2) = 3$	124	Yes
ES with a rearrangement of output arcs of monitors	$12p_9 + p_{10} + 8p_{11}$	$11v_1 + 5v_2$	$W(v_1, t_1) = W(t_2, v_1) = 2$, $W(v_1, t_6) = W(t_7, v_1) = 1$, $W(v_2, t_2) = W(t_3, v_2) = 1$, $W(v_2, t_5) = W(t_6, v_2) = 3$	168	Yes
ILS by removing 1 token from p_9	$11p_9 + p_{10} + 8p_{11}$	/	/	206	Yes
ILS by adding 1 token in p_9	$13p_9 + p_{10} + 8p_{11}$	/	/	242	Yes

The meaning of the work:

- Extend the ILS-based method to a more general subclass of Petri nets;
- Extend the concepts of WSDCs and competition paths;
- Provide a deeper insight into structures of more general Petri nets.

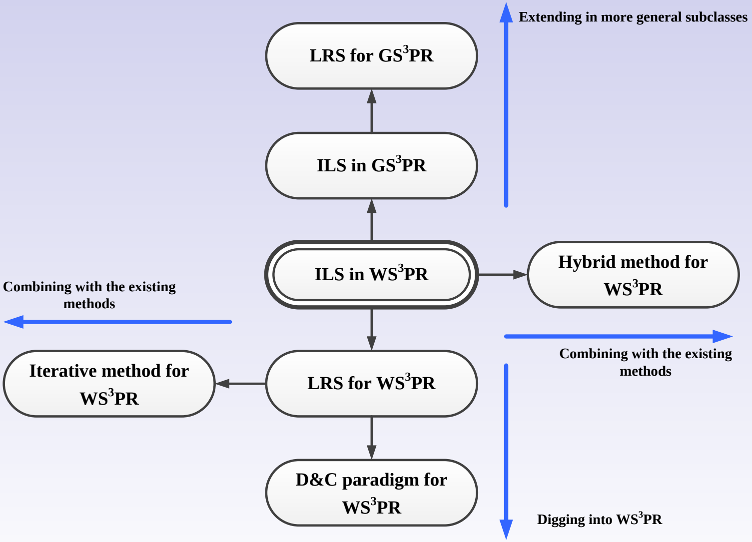


Figure: Main work.

Conclusion:

- Numerical relationship in generalized Petri nets;
- Intrinsically live structure – ILS;
- Liveness and ratio-enforcing supervisor – LRS.

Related publications:

[1] **Ding Liu**, ZhiWu Li, and MengChu Zhou, "Liveness of an extended S^3PR," *Automatica*, vol. 46, no. 6, pp. 1008–1018, 2010.

[2] **Ding Liu**, ZhiWu Li, and MengChu Zhou, "Erratum to "Liveness of an extended S^3PR [*Automatica* 46 (2010) 1008–1018]"," *Automatica*, vol. 48, no. 5, pp. 1003–1004, 2012.

[3] **Ding Liu**, ZhiWu Li, and MengChu Zhou, "Hybrid liveness-enforcing policy for generalized Petri net models of flexible manufacturing systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 43, no. 1, pp. 85–97, 2013.

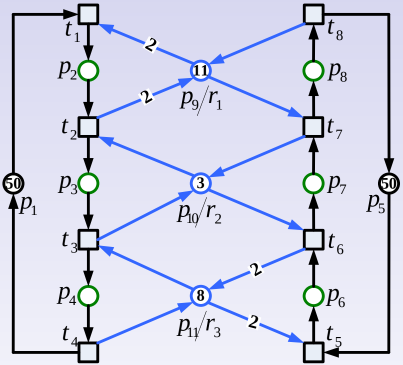
[4] **Ding Liu**, Kamel Barkaoui, and MengChu Zhou, "On intrinsically live structure of a class of generalized Petri nets modeling FMS," in *Proceedings of the 11th International Workshop on Discrete Event Systems*, pp. 187–192, Guadalajara, Mexico, Oct. 3-5, 2012.

[5] **Ding Liu**, YiFan Hou, Kamel Barkaoui, and MengChu Zhou, "Liveness and resource usage ratio-enforcing supervisor in a class of generalized Petri nets," submitted to the 10th International Conference on Control and Automation, Hangzhou, China, Jun. 12-14 2013.

Future work:

- Characteristics of ILS in state space;
- Optimal design of LRS;
- ILS in ROPN.

Work in progress:



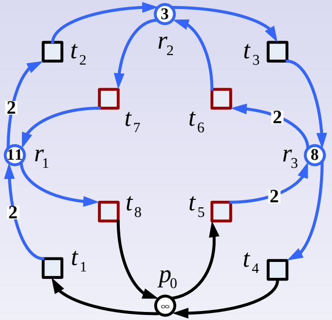
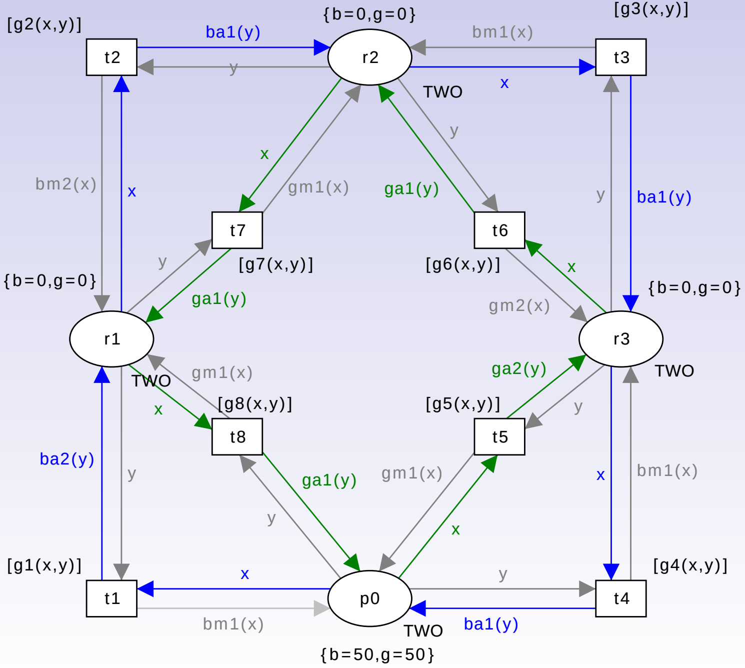


Figure: A process-oriented Petri net model and a resource-oriented one.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{b=0,g=0}
[g3(x,y)]
t2
ba1(y)
r2
bm1(x)
t3
y
x
TWO
x
y
bm2(x)
x
gm1(x)
ga1(y)
y
ba1(y)
t7
t6
y
[g7(x,y)]
[g6(x,y)]
x
{b=0,g=0}
ga1(y)
gm2(x)
{b=0,g=0}
r1
r3
gm1(x)
ga2(y)
TWO
TWO
x
[g8(x,y)]
[g5(x,y)]
y
t8
t5
ba2(y)
y
ga1(y)
gm1(x)
x
bm1(x)
y
x
[g1(x,y)]
x
y
[g4(x,y)]
t1
p0
t4
bm1(x)
TWO
ba1(y)
{b=50,g=50}

Thanks for your attention!

Questions?

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