# Reachability in Timed Counter Systems

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☞ *We combine Timed Automata and Counter Systems and we study their reachability matters*

# Outline

1. **Timed Counter Systems**
   - Example
   - Definitions
   - Semantics
2. **Reachability**
   - Counter Reachability Problem
3. **Analysis of TCS via clock abstraction**
   - Region Graph construction
   - The Region Graph as a Counter System
4. **Subclasses of TCS**
   - Decidability results
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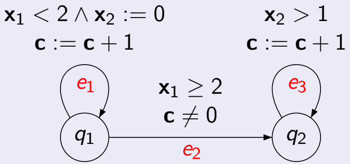
# Example

## a Timed Counter System



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# Definitions

$X$ = a set of $m$ real-valued variables, called clocks.

$\mathbf{x}$ = a valuation of the clocks, in $\mathbb{R}_+^m$.

$R_X$ = the set of relations on clocks

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$C$ = a set of $n$ integer-valued variables, called counters.
$\mathbf{c}$ = a valuation of the counters, in $\mathbb{Z}^n$.
$R_C$ = the set of relations on counters
☞Presburger-definable binary relations ($\equiv$ semi-linear)

# Definitions (continued)

**Definition**

A Timed Counter System is a tuple $\langle Q, X, C, E \rangle$ where :

- $Q$ is a finite set of control states (also called *locations*)
- $E \subseteq Q \times R_X \times R_C \times Q$ is a finite set of transitions (edges)

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**Definition**

A *Timed Automaton* is a TCS where $C = \emptyset$.
A *Counter System* is a TCS where $X = \emptyset$.

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- Counting Transition System $CTS(\mathcal{S})$
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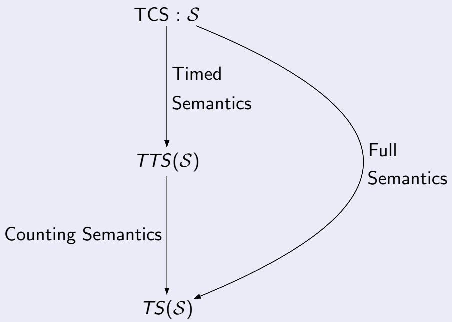
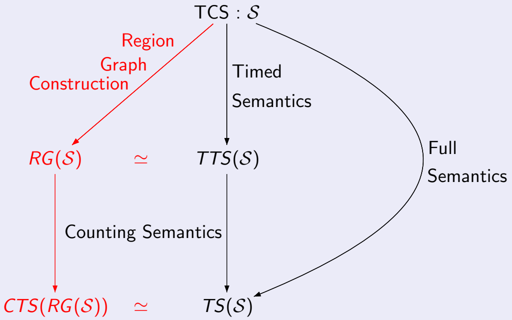


# The different semantics of a TCS $\mathcal{S}$

- Counting Transition System $CTS(\mathcal{S})$
- Timed Transition System $TTS(\mathcal{S})$ $\simeq$ Region Graph $RG(\mathcal{S})$
- full Transition System $TS(\mathcal{S})$ $\simeq$ $CTS(RG(\mathcal{S}))$



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Clocks are used for modelling temporal requirements ; their *exact* value does not really matter.

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## Counter Reachability Problem (CRP)

**Inputs :** A TCS $\mathcal{S}$, an initial configuration $s_0$ of $TS(\mathcal{S})$, and a configuration $(q, \mathbf{c})$ of $CTS(\mathcal{S})$.

**Question :** Is there a clock valuation $\mathbf{x}$ such that $(q, \mathbf{x}, \mathbf{c})$ is reachable from $s_0$ in $TS(\mathcal{S})$ ?

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The CRP extends the classical reachability problem of CS, known to be undecidable ; therefore CRP is undecidable for TCS.

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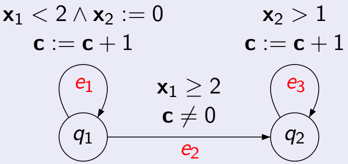

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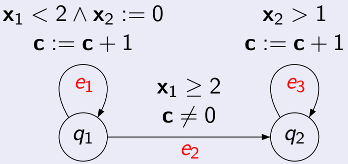
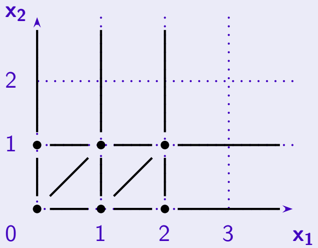


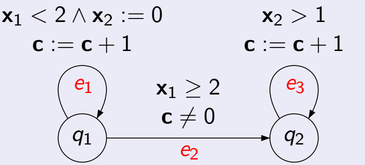

## ...and its clock Regions



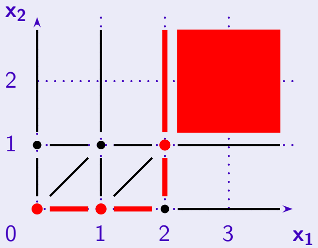

28 regions in total : 6 points, 9 line segments, 5 half-lines, 4 triangular closed areas, and 4 open areas

# Example

## a Timed Counter System...



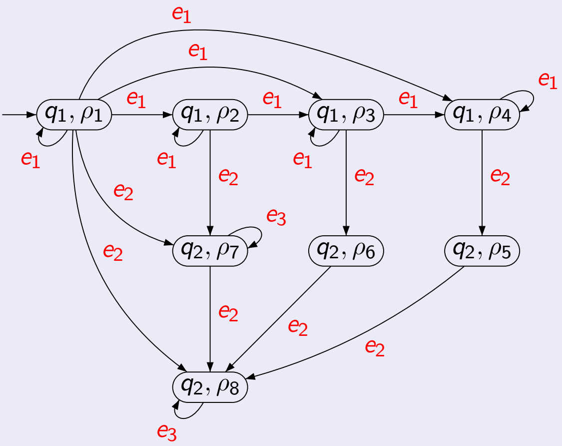

## ...and its reachable Regions




8 reachable regions (out of 28), considering the initial configuration $\left(q_1, \binom{0}{0}, 0\right)$

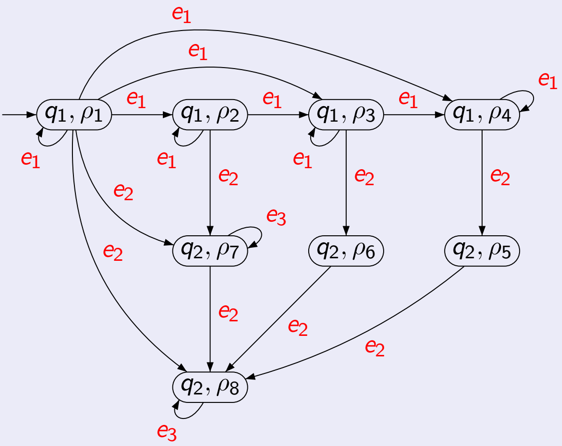
# Example (continued)

## ...and its Region Graph



# Example (continued)

## ...and its Region Graph which is a Counter System !



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**Key idea :**

For a TCS $\mathcal{S}$, its region graph $RG(\mathcal{S})$ is also a Counter System (namely because it has a finite number of states).

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Let $\mathfrak{C}$ be a class of TCS such that there is an algorithm solving the classical reachability problem for $RG(\mathcal{S})$, for any $\mathcal{S} \in \mathfrak{C}$.

**Theorem**

The Counter Reachability Problem is decidable for $\mathfrak{C}$.

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**Theorem**

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**Proof idea** ▸ time-abstract bisimulation

By definition, $CTS(TTS(\mathcal{S})) = TTS(CTS(\mathcal{S})) = TS(\mathcal{S})$.
It is well-known that $RG(\mathcal{S}) \simeq TTS(\mathcal{S})$.
Therefore $CTS(RG(\mathcal{S})) \simeq TS(\mathcal{S})$. □

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# Subclasses of TCS

- Timed Counter Machine (TCM) $=$ TCS whose relations on counters are translations with guards of the form $b \leq \mathbf{c}$ or $b = \mathbf{c}$, where $b \in \mathbb{N}^n$
- Timed VASS (TVASS) $=$ TCM without $b = \mathbf{c}$ guards
- Bounded TCS $=$ TCS whose counter values are bounded
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## Decidability results

| Model | Region Graph | Counter Reachability |
|---|---|---|
| TCS | CS | Undecidable |
| TVASS | VASS | Decidable |
| Reversal-bounded TCM | Reversal-bounded CM | Decidable |
| Bounded TCS | Bounded CS | Decidable |

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# Algorithm solving the CRP

Since TVASS is a recursive class, we propose an algorithm solving the CRP for this class :

**Inputs :** A TVASS $\mathcal{S}$, a configuration $(q, \mathbf{c})$, and an initial state $s_0$
**Output :** Answers "Is there a $\mathbf{x}$ such that $(q, \mathbf{x}, \mathbf{c})$ is reachable from $s_0$ in $TS(\mathcal{S})$ ?"

1. Build $RG(\mathcal{S})$
2. For all state $(q', [\mathbf{x}])$ of $RG(\mathcal{S})$ do
3.     If $q' = q$ then
4.         If $((q, [\mathbf{x}]), \mathbf{c})$ is reachable in $RG(\mathcal{S})$ from $s_0$ then
5.             return *True*
6. return *False*

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- Decidability results for CRP on 3 subclasses of TCS

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- Broaden decidability results : flat TCS, etc...
- Extend the tool FAST [BFLP03] with time
- Generalize our main theorem to other datatypes than counters : pushdown stacks, lossy channels, etc...

# Related work

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- real-valued counters [DIPX04, XDIP03]

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